

Lateral-Directional Eigenvector Flying Qualities Guidelines for High Performance Aircraft

John B. Davidson
NASA Langley Research Center, Hampton, Virginia

Dominick Andrisani, II
Purdue University, West Lafayette, Indiana

December 1996

National Aeronautics and
Space Administration
Langley Research Center
Hampton, Virginia 23681-0001

SUMMARY

This report presents the development of lateral-directional flying qualities guidelines with application to eigenspace (eigenstructure) assignment methods. These guidelines will assist designers in choosing eigenvectors to achieve desired closed-loop flying qualities or performing trade-offs between flying qualities and other important design requirements, such as achieving realizable gain magnitudes or desired system robustness. This has been accomplished by developing relationships between the system's eigenvectors and the roll rate and sideslip transfer functions. Using these relationships, along with constraints imposed by system dynamics, key eigenvector elements are identified and guidelines for choosing values of these elements to yield desirable flying qualities have been developed. Two guidelines are developed - one for low roll-to-sideslip ratio and one for moderate-to-high roll-to-sideslip ratio. These flying qualities guidelines are based upon the Military Standard lateral-directional coupling criteria for high performance aircraft - the roll rate oscillation criteria and the sideslip excursion criteria. Example guidelines are generated for a moderate-to-large, an intermediate, and low value of roll-to-sideslip ratio.

1.0 INTRODUCTION

The Direct Eigenspace Assignment (DEA) method (Davidson and Schmidt 1986) is currently being used to design lateral-directional control laws for NASA's High Angle-of-Attack Research Vehicle (HARV) (Davidson et al. 1992). This method allows designers to shape the closed-loop response by judicious choice of desired eigenvalues and eigenvectors. During this design effort DEA has been demonstrated to be a useful technique for aircraft control design. The control laws developed using this method have demonstrated good performance, robustness, and flying qualities during both piloted simulation and flight testing (Murphy et al. 1994).

During the control law design effort, two limitations of this method became apparent. First, when using DEA the designer has no direct control over augmentation gain magnitudes. Often it is not clear how to adjust the desired eigenspace in order to reduce individual undesirable gain magnitudes. Second, although considerable guidance is available for choosing desired eigenvalues (Military Standard, time constants, frequency, and damping specifications), little guidance is available for choosing desired system eigenvectors. Design guidance is needed on how to select closed-loop lateral-directional eigenvectors to achieve desired flying qualities.

The first limitation was addressed by the development of Gain Weighted Eigenspace Assignment (GWEA) (Davidson and Andrisani 1994). The GWEA method allows a designer to place eigenvalues at desired locations and trade-off the achievement of desired eigenvectors versus feedback gain magnitudes. This report addresses the second limitation by presenting the development of lateral-directional flying qualities guidelines with application to eigenspace assignment methods. These guidelines will assist designers in choosing eigenvectors to achieve desired closed-loop flying qualities or performing trade-offs between flying qualities and other important design requirements, such as achieving realizable gain magnitudes or desired system robustness.

This report is organized into four sections. A review of lateral/directional dynamics, background information on how eigenvalues and eigenvectors influence a system's dynamic response, a review of the Direct Eigenspace Assignment methodology, and an overview of existing lateral/directional flying qualities criteria is presented in the following section. The development of the lateral-directional eigenvector flying qualities guidelines are presented in the third section. Concluding remarks are given in the final section.

2.0 BACKGROUND

This section presents a review of lateral/directional dynamics, background information on how eigenvalues and eigenvectors influence a system's dynamic response, a review of the Direct Eigenspace Assignment methodology, and an overview of existing lateral/directional flying qualities criteria.

Lateral-Directional Dynamics

The linearized rigid body lateral-directional equations of motion for a steady, straight, and level flight condition, referenced to stability axes, are (McRuer et al. 1973)

$$\begin{bmatrix} Y_{\beta} + (Y_{\dot{\beta}} - 1)s & \frac{g}{V_0} + (Y_p + \alpha_0)s & (Y_r - 1) \\ L_{\beta} & -(s - L_p)s & L_r \\ N_{\beta} & N_p s & -(s - N_r) \end{bmatrix} \begin{bmatrix} \beta \\ \phi \\ r \end{bmatrix} = - \begin{bmatrix} Y_{\delta_{ail}} & Y_{\delta_{rud}} \\ L_{\delta_{ail}} & L_{\delta_{rud}} \\ N_{\delta_{ail}} & N_{\delta_{rud}} \end{bmatrix} \begin{bmatrix} \delta_{ail} \\ \delta_{rud} \end{bmatrix}$$

$$p = s\phi \quad (2.1a)$$

or in state space form

$$\frac{d}{dt} \begin{bmatrix} \beta \\ p \\ r \\ \phi \end{bmatrix} = \begin{bmatrix} Y_{\beta}/(1 - Y_{\dot{\beta}}) & (Y_p + \alpha_0)/(1 - Y_{\dot{\beta}}) & (Y_r - 1)/(1 - Y_{\dot{\beta}}) & (g/V_0)/(1 - Y_{\dot{\beta}}) \\ L_{\beta} & L_p & L_r & 0 \\ N_{\beta} & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ p \\ r \\ \phi \end{bmatrix} + \begin{bmatrix} Y_{\delta_{ail}}/(1 - Y_{\dot{\beta}}) & Y_{\delta_{rud}}/(1 - Y_{\dot{\beta}}) \\ L_{\delta_{ail}} & L_{\delta_{rud}} \\ N_{\delta_{ail}} & N_{\delta_{rud}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{ail} \\ \delta_{rud} \end{bmatrix} \quad (2.1b)$$

where

- β = sideslip angle
- p = stability axis roll rate
- r = stability axis yaw rate
- ϕ = bank angle
- δ_{ail} = aileron control input
- δ_{rud} = rudder control input

and the prime denotes the inclusion of the inertia terms. As can be seen, the lateral (p) and directional (β and r) responses are coupled. The primary lateral-directional coupling derivatives are: roll moment due to sideslip angle L_{β} , roll moment due to yaw rate L_r , yaw moment due to roll rate N_p , and yaw moment due to lateral controls N_{δ} . A brief review of the physical basis of these derivatives is given in the Appendix.

The characteristic equation for this system is

$$\begin{aligned}
\Delta(s) = & \left[Y_{\dot{\beta}} - 1 \right] s^4 + \left[Y_{\beta} - \left(Y_{\dot{\beta}} - 1 \right) \left(N_r' + L_p' \right) \right] s^3 \\
& + \left[-Y_{\beta} \left(N_r' + L_p' \right) + \left(Y_{\dot{\beta}} - 1 \right) \left(N_r' L_p' - L_r' N_p' \right) + N_{\beta}' (Y_r - 1) + L_{\beta}' (Y_p + \alpha_0) \right] s^2 \\
& + \left[Y_{\beta} \left(N_r' L_p' - L_r' N_p' \right) + \left(L_{\beta}' N_p' - L_p' N_{\beta}' \right) (Y_r - 1) \right. \\
& + \left. \left(N_{\beta}' L_r' - L_{\beta}' N_r' \right) (Y_p + \alpha_0) + (g/V_0) L_{\beta}' \right] s \\
& + (g/V_0) \left[N_{\beta}' L_r' - L_{\beta}' N_r' \right]
\end{aligned} \tag{2.2}$$

There are three classical lateral-directional eigenvalues: a lightly damped oscillatory pole referred to as the Dutch roll pole (λ_{dr}), a first order pole with a long time constant referred to as the spiral pole (λ_{sprl}), and a first order pole with a relatively short time constant referred to as the roll pole (λ_{roll}). The characteristic equation can be written in terms of these eigenvalues as

$$\begin{aligned}
\Delta(s) &= k_{\Delta} (s - \lambda_{sprl}) (s - \lambda_{roll}) (s^2 + 2\zeta_{dr} \omega_{dr} s + \omega_{dr}^2) \\
&= k_{\Delta} (s - \lambda_{sprl}) (s - \lambda_{roll}) (s - \lambda_{dr}) (s - \bar{\lambda}_{dr})
\end{aligned} \tag{2.3}$$

where $k_{\Delta} = Y_{\dot{\beta}} - 1$, $\lambda_{dr} = -\omega_{dr} \zeta_{dr} + j \omega_{dr} \sqrt{1 - \zeta_{dr}^2}$ and $\bar{\lambda}_{dr}$ denotes the complex conjugate of λ_{dr} . Approximations for the system eigenvalues in terms of stability and control derivatives (McRuer et al. 1973) are given by:

$$\omega_{dr}^2 \cong N_{\beta}' \tag{2.4}$$

$$2\omega_{dr} \zeta_{dr} \cong -N_r' - Y_{\beta} - \left(\frac{L_{\beta}'}{N_{\beta}'} \right) \left(N_p' - \frac{g}{V_0} \right) \tag{2.5}$$

$$\lambda_{roll} \cong L_p' - \left(\frac{L_{\beta}'}{N_{\beta}'} \right) \left(N_p' - \frac{g}{V_0} \right) \tag{2.6}$$

$$\lambda_{sprl} \cong \frac{g}{\lambda_{roll} V_0} \left(N_r' \left(\frac{L_{\beta}'}{N_{\beta}'} \right) - L_r' \right) \tag{2.7}$$

The primary lateral-directional control task is control of bank angle with lateral stick. The following relationships are developed for lateral stick controlling aileron deflection ($\delta_{stk} = \delta_{ail}$) with zero rudder input. In the following, the sub-subscript “ail” on the control derivatives has been dropped to simplify the notation. The bank angle-to-lateral stick transfer function is given by

$$\begin{aligned} \frac{\phi}{\delta_{stk}} = \frac{-1}{\Delta(s)} & \left\{ \left[-L_{\delta}'(Y_{\dot{\beta}} - 1) \right] s^2 \right. \\ & + \left[Y_{\delta}' L_{\beta}' - N_{\delta}' L_r' (Y_{\dot{\beta}} - 1) + L_{\delta}' (N_r' (Y_{\dot{\beta}} - 1) - Y_{\beta}') \right] s \\ & \left. + \left[Y_{\delta}' (N_{\beta}' L_r' - L_{\beta}' N_r') + N_{\delta}' (L_{\beta}' (Y_r - 1) - L_r' Y_{\beta}') + L_{\delta}' (N_r' Y_{\beta}' - N_{\beta}' (Y_r - 1)) \right] \right\} \end{aligned} \quad (2.8)$$

This transfer function can be written in pole-zero form as

$$\frac{\phi}{\delta_{stk}} = \frac{L_{\delta}' (s^2 + 2\zeta_{\phi} \omega_{\phi} s + \omega_{\phi}^2)}{(s - \lambda_{sprl})(s - \lambda_{roll})(s^2 + 2\zeta_{dr} \omega_{dr} s + \omega_{dr}^2)} \quad (2.9)$$

The following relationships can be written from (2.8) and (2.9)

$$\begin{aligned} \omega_{\phi}^2 \cong \frac{1}{1 - Y_{\dot{\beta}}'} & \left\{ N_r' Y_{\beta}' - N_{\beta}' (Y_r - 1) + \left(\frac{N_{\delta}'}{L_{\delta}'} \right) (L_{\beta}' (Y_r - 1) - L_r' Y_{\beta}') \right. \\ & \left. + \left(\frac{Y_{\delta}'}{N_{\delta}'} \right) (N_{\beta}' L_r' - L_{\beta}' N_r') \right\} \end{aligned} \quad (2.10)$$

and

$$2\zeta_{\phi} \omega_{\phi} = \frac{1}{1 - Y_{\dot{\beta}}'} \left\{ N_r' (Y_{\dot{\beta}} - 1) - Y_{\beta}' - \left(\frac{N_{\delta}'}{L_{\delta}'} \right) L_r' (Y_{\dot{\beta}} - 1) + \left(\frac{Y_{\delta}'}{N_{\delta}'} \right) L_{\beta}' \right\} \quad (2.11)$$

By making the following assumptions (reasonable for most configurations (McRuer et al. 1973))

$$Y_r \cong 0, Y_{\dot{\beta}} \cong 0, Y_{\delta} \cong 0 \quad (2.12a)$$

$$\left| Y_{\beta}' (N_r' L_{\delta}' - N_{\delta}' L_r') \right| \ll \left| N_{\beta}' L_{\delta}' - N_{\delta}' L_{\beta}' \right| \quad (2.12b)$$

equations (2.10) and (2.11) reduce to

$$\omega_\phi^2 \equiv N_\beta' \left(1 - \left(\frac{N_\delta'}{L_\delta'} \right) \left(\frac{L_\beta'}{N_\beta'} \right) \right) \quad (2.13)$$

$$2\omega_\phi \zeta_\phi \equiv -N_r' - Y_\beta + \left(\frac{N_\delta'}{L_\delta'} \right) L_r' \quad (2.14)$$

Since $p=s\phi$, the roll rate-to-lateral stick transfer function can be written

$$\frac{p}{\delta_{stk}} = \frac{s\phi}{\delta_{stk}} = \frac{L_\delta' s (s^2 + 2\zeta_\phi \omega_\phi s + \omega_\phi^2)}{(s - \lambda_{sprl})(s - \lambda_{roll})(s^2 + 2\zeta_{dr} \omega_{dr} s + \omega_{dr}^2)} \quad (2.15)$$

The steady-state roll rate for a unit step lateral stick input (assuming the spiral pole is approximately at the origin) is given by

$$p_{ss} = L_\delta' \left(\frac{-1}{\lambda_{roll}} \right) \left(\frac{\omega_\phi}{\omega_{dr}} \right)^2 \quad (2.16)$$

The sideslip-to-lateral stick transfer function is given by

$$\begin{aligned} \frac{\beta}{\delta_{stk}} = \frac{-1}{\Delta(s)} & \left\{ Y_\delta s^3 + \left[-Y_\delta (N_r' + L_p') + N_\delta' (Y_r - 1) + L_\delta' (Y_p + \alpha_0) \right] s^2 \right. \\ & + \left[Y_\delta (N_r' L_p' - L_r' N_p') - N_\delta' L_p' (Y_r - 1) + N_\delta' L_r' (Y_p + \alpha_0) \right. \\ & \left. \left. + L_\delta' N_p' (Y_r - 1) + L_\delta' \left(\frac{g}{V_0} - N_r' (Y_p + \alpha_0) \right) \right] s \right. \\ & \left. + \frac{g}{V_0} \left[N_\delta' L_r' - L_\delta' N_r' \right] \right\} \end{aligned} \quad (2.17)$$

Making the assumptions of (2.12a), and that the spiral pole is close to the origin, and that

$$Y_p + \alpha_0 \cong 0 \quad (2.18a)$$

$$\frac{g}{V_0} \left[N_\delta' L_r' - L_\delta' N_r' \right] \cong 0 \quad (2.18b)$$

this transfer function can be written as

$$\frac{\beta}{\delta_{stk}} = \frac{-N_\delta' \left(s + \left(-L_p' + \frac{L_\delta'}{N_\delta'} \left(N_p' - \frac{g}{V_0} \right) \right) \right)}{(s - \lambda_{roll})(s^2 + 2\zeta_{dr} \omega_{dr} s + \omega_{dr}^2)} \quad (2.19)$$

The yaw rate-to-lateral stick transfer function is given by

$$\begin{aligned} \frac{r}{\delta_{stk}} = \frac{-1}{\Delta(s)} & \left\{ -N'_\delta(Y_{\dot{\beta}} - 1)s^3 + \left[Y_\delta N'_\beta + N'_\delta(\dot{L}_p(Y_{\dot{\beta}} - 1) - Y_\beta) - \dot{L}_\delta N'_p(Y_{\dot{\beta}} - 1) \right] s^2 \right. \\ & + \left[-Y_\delta(N'_\beta \dot{L}_p - \dot{L}_\beta N'_p) + N'_\delta(Y_\beta \dot{L}_p - \dot{L}_\beta(Y_p + \alpha_0)) \right. \\ & \quad \left. \left. - \dot{L}_\delta(Y_\beta N'_p - N'_\beta(Y_p + \alpha_0)) \right] s \right. \\ & \left. + \frac{g}{V_0} [\dot{L}_\delta N'_\beta - N'_\delta \dot{L}_\beta] \right\} \end{aligned} \quad (2.20)$$

Making the assumptions of (2.12a) and (2.18a), this transfer function can be written as

$$\begin{aligned} \frac{r}{\delta_{stk}} = \frac{-1}{\Delta(s)} & \left\{ N'_\delta s^3 + \left[-N'_\delta(\dot{L}_p + Y_\beta) + \dot{L}_\delta N'_p \right] s^2 \right. \\ & \left. + \left[Y_\beta(N'_\delta \dot{L}_p - \dot{L}_\delta N'_p) \right] s + \frac{g}{V_0} [\dot{L}_\delta N'_\beta - N'_\delta \dot{L}_\beta] \right\} \end{aligned} \quad (2.21)$$

Eigenvalues, Eigenvectors, and System Dynamic Response

The eigenvalues and eigenvectors of a system are related to its dynamic response in the following way. Given the observable and controllable linear time-invariant system

$$\dot{x} = Ax + Bu \quad (2.22a)$$

and output equation

$$y = Cx \quad (2.22b)$$

where $x \in \mathbf{R}^n$, $u \in \mathbf{R}^m$, and $y \in \mathbf{R}^l$.

The Laplace transform of equation (2.22a) is given by

$$sx(s) - x(0) = Ax(s) + Bu(s) \quad (2.23a)$$

$$x(s) = [sI_n - A]^{-1}x(0) + [sI_n - A]^{-1}Bu(s) \quad (2.23b)$$

Solution of equation (2.22a) is given by taking the inverse Laplace Transform of equation (2.23b)

$$x(t) = \mathcal{L}^{-1}\{[sI_n - A]^{-1}\}x(0) + \mathcal{L}^{-1}\{[sI_n - A]^{-1}Bu(s)\} \quad (2.24)$$

Noting that

$$\mathcal{L}^{-1}\{[sI_n - A]^{-1}\} = e^{At} \quad (2.25)$$

the solution of (2.24) is (Brogan 1974)

$$x(t) = e^{At} x(0) + \int_0^t e^{A(t-\tau)} B u(\tau) d\tau \quad (2.26)$$

and system outputs are

$$y(t) = C e^{At} x(0) + \int_0^t C e^{A(t-\tau)} B u(\tau) d\tau \quad (2.27)$$

The system dynamic matrix, A , can be represented by

$$A = V \Lambda V^{-1} = V \Lambda L \quad (2.28)$$

where V is a matrix of system eigenvectors, L is the inverse eigenvector matrix, and Λ is a diagonal matrix of system eigenvalues. Given this result, e^{At} can be expressed by

$$e^{At} = V e^{\Lambda t} L = \sum_{j=1}^n v_j e^{\lambda_j t} l_j \quad (2.29)$$

where λ_j is the j^{th} system eigenvalue, v_j is the j^{th} column of V (j^{th} eigenvector of A), and l_j is the j^{th} row of L (j^{th} left eigenvector of A). Equation (2.27) can then be expressed as

$$y(t) = \sum_{j=1}^n C v_j e^{\lambda_j(t)} l_j x(0) + \sum_{j=1}^n C v_j \int_0^t e^{\lambda_j(t-\tau)} l_j B u(\tau) d\tau \quad (2.30)$$

Noting that

$$B u(t) = \sum_{k=1}^m b_k u_k(t) \quad (2.31)$$

where b_k is the k^{th} column of B and u_k is the k^{th} system input, the system outputs due to initial conditions and input u_k is given by

$$y(t) = \sum_{j=1}^n C v_j e^{\lambda_j(t)} l_j x(0) + \sum_{j=1}^n \sum_{k=1}^m C v_j l_j b_k \int_0^t e^{\lambda_j(t-\tau)} u_k(\tau) d\tau \quad (2.32)$$

The i^{th} system output is given by

$$y_i(t) = \sum_{j=1}^n c_i v_j e^{\lambda_j(t)} l_j x(0) + \sum_{j=1}^n \sum_{k=1}^m c_i v_j l_j b_k \int_0^t e^{\lambda_j(t-\tau)} u_k(\tau) d\tau \quad (2.33)$$

where c_i is the i^{th} row of C . In the case of initial conditions equal to zero, the i^{th} output is given by

$$y_i(t) = \sum_{j=1}^n \sum_{k=1}^m R_{i,j,k} \int_0^t e^{\lambda_j(t-\tau)} u_k(\tau) d\tau \quad (2.34)$$

where $R_{i,j,k} = c_i v_j l_j b_k$. In this expression, $R_{i,j,k}$ is the modal residue for output i , associated with eigenvalue j , and due to input k .

Given an impulsive input in the k^{th} input, equation (2.34) reduces to

$$y_i(t) = \sum_{j=1}^n \sum_{k=1}^m R_{i,j,k} e^{\lambda_j(t)} \quad (2.35)$$

and for a step input in the k^{th} input, equation (2.34) reduces to (for $\lambda_j \neq 0$)

$$y_i(t) = \sum_{j=1}^n \sum_{k=1}^m \left(\frac{R_{i,j,k}}{\lambda_j} \right) e^{\lambda_j(t)} \quad (2.36)$$

As these expressions show, a system's dynamics are dependent on both its eigenvalues and its eigenvectors. The eigenvalues determine the time constant or frequency and damping of each mode. The eigenvectors determine the residues. The residues determine how much each mode of the system contributes to a given output.

For example, for the lateral-directional system given by equation (2.1), time responses for a unit step lateral stick input (and zero pedal input) can be written in terms of system eigenvalues and residues as (because there is only one input, the third subscript on the R 's has been omitted)

$$\begin{aligned} p(t) &= \frac{R_{p,sprl}}{\lambda_{sprl}} e^{\lambda_{sprl}t} + \frac{R_{p,roll}}{\lambda_{roll}} e^{\lambda_{roll}t} + 2 \left| \frac{R_{p,dr}}{\lambda_{dr}} \right| e^{-\zeta_{dr}\omega_{dr}t} \cos(\omega_{dr}\sqrt{1-\zeta_{dr}^2}t + \Psi_p) \\ \beta(t) &= \beta_0 + \frac{R_{\beta,sprl}}{\lambda_{sprl}} e^{\lambda_{sprl}t} + \frac{R_{\beta,roll}}{\lambda_{roll}} e^{\lambda_{roll}t} \\ &\quad + 2 \left| \frac{R_{\beta,dr}}{\lambda_{dr}} \right| e^{-\zeta_{dr}\omega_{dr}t} \cos(\omega_{dr}\sqrt{1-\zeta_{dr}^2}t + \Psi_\beta) \\ r(t) &= \frac{R_{r,sprl}}{\lambda_{sprl}} e^{\lambda_{sprl}t} + \frac{R_{r,roll}}{\lambda_{roll}} e^{\lambda_{roll}t} + 2 \left| \frac{R_{r,dr}}{\lambda_{dr}} \right| e^{-\zeta_{dr}\omega_{dr}t} \cos(\omega_{dr}\sqrt{1-\zeta_{dr}^2}t + \Psi_r) \end{aligned} \quad (2.37)$$

with

$$\beta_0 = - \left(\frac{R_{\beta,sprl}}{\lambda_{sprl}} + \frac{R_{\beta,roll}}{\lambda_{roll}} + 2 \left| \frac{R_{\beta,dr}}{\lambda_{dr}} \right| \cos(\angle \frac{R_{\beta,dr}}{\lambda_{dr}}) \right) \quad (2.38)$$

and

$$\Psi_p = \angle \frac{R_{p,dr}}{\lambda_{dr}}; \quad \Psi_\beta = \angle \frac{R_{\beta,dr}}{\lambda_{dr}}; \quad \Psi_r = \angle \frac{R_{r,dr}}{\lambda_{dr}} \quad (2.39)$$

where $|x|$ denotes the magnitude of x and $\angle x$ denotes the phase angle of x .

Direct Eigenspace Assignment Methodology

One possible approach to the aircraft control synthesis problem would be to synthesize a control system that would control both the eigenvalue locations and the residue magnitudes. Since the residues are a function of the system's eigenvectors this naturally leads to a control synthesis technique that involves achieving some desired eigenspace in the closed-loop system (eigenspace (eigenstructure) assignment) (Moore 1976; Srinathkumar 1978; Cunningham 1980; Andry 1983). An eigenspace assignment method currently being used to design control laws for NASA's High Angle-of-attack Research Vehicle (HARV) is Direct Eigenspace Assignment (DEA) (Davidson et al. 1992; Murphy et al. 1994). DEA is a control synthesis technique for directly determining measurement feedback control gains that will yield an achievable eigenspace in the closed-loop system. For a system that is observable and controllable and has n states, m controls, and l measurements; DEA will determine a gain matrix that will place l eigenvalues to desired locations and m elements of their associated eigenvectors to desired values[†]. If it is desired to place more than m elements of the associated l eigenvectors, DEA yields eigenvectors in the closed-loop system that are as close as possible in a least squares sense to desired eigenvectors. A more detailed development can be found in Davidson and Schmidt, 1986.

Direct Eigenspace Assignment Formulation

Given the observable, controllable system

$$\dot{x} = Ax + Bu \quad (2.40a)$$

where $x \in \mathbf{R}^n$ and $u \in \mathbf{R}^m$, with system measurements given by

$$z = Mx + Nu \quad (2.40b)$$

where $z \in \mathbf{R}^l$.

The total control input is the sum of the augmentation input u_c and pilot's input u_p

$$u = u_p + u_c \quad (2.41)$$

The measurement feedback control law is

$$u_c = Gz \quad (2.42)$$

Solving for u as a function of the system states and pilot's input yields

$$u = [I_m - GN]^{-1} GMx + [I_m - GN]^{-1} u_p \quad (2.43)$$

The system augmented with the control law is given by

$$\dot{x} = (A + B[I_m - GN]^{-1} GM)x + B[I_m - GN]^{-1} u_p \quad (2.44)$$

The spectral decomposition of the closed-loop system is given by

$$(A + B[I_m - GN]^{-1} GM)v_i = \lambda_i v_i \quad (2.45)$$

[†] This assumes $l > m$. For a general statement and proof of this property the reader is referred to Srinathkumar 1978.

for $i = 1, \dots, n$ where λ_i is the i^{th} system eigenvalue and v_i is the associated i^{th} system eigenvector. Let w_i be defined by

$$w_i \equiv [I_m - GN]^{-1} GM v_i \quad (2.46)$$

Substituting this result into equation (2.45) and solving for v_i yields

$$v_i = [\lambda_i I_n - A]^{-1} B w_i \quad (2.47)$$

This equation describes the achievable i^{th} eigenvector of the closed-loop system as a function of the eigenvalue λ_i and w_i . By examining this equation, one can see that the number of control variables (m) determines the dimension of the subspace in which the achievable eigenvectors must reside.

Values of w_i that yield an achievable eigenspace that is as close as possible in a least squares sense to a desired eigenspace can be determined by defining a cost function associated with the i^{th} mode of the system

$$J_i = \frac{1}{2} (v_{a_i} - v_{d_i})^H Q_{d_i} (v_{a_i} - v_{d_i}) \quad (2.48)$$

for $i = 1, \dots, l$ where v_{a_i} is the i^{th} achievable eigenvector associated with eigenvalue λ_i , v_{d_i} is the i^{th} desired eigenvector, and Q_{d_i} is an n -by- n symmetric positive semi-definite weighting matrix on eigenvector elements[†]. This cost function represents the error between the achievable eigenvector and the desired eigenvector weighted by the matrix Q_{d_i} .

Values of w_i that minimize J_i are determined by substituting (2.47) into the cost function for v_{a_i} , taking the gradient of J_i with respect to w_i , setting this result equal to zero, and solving for w_i . This yields

$$w_i = [A_{d_i}^H Q_{d_i} A_{d_i}]^{-1} A_{d_i}^H Q_{d_i} v_{d_i} \quad (2.49)$$

where

$$A_{d_i} = [\lambda_{d_i} I_n - A]^{-1} B \quad (2.50)$$

and λ_{d_i} is the i^{th} desired eigenvalue of the closed-loop system. Note in this development λ_{d_i} cannot belong to the spectrum of A .

By concatenating the individual w_i 's column-wise to form W and v_{a_i} 's column-wise to form V_a , equation (2.46) can be expressed in matrix form by

$$W = [I_m - GN]^{-1} GM V_a \quad (2.51)$$

[†] Superscript H denotes complex conjugate transpose (Strang 1980).

From (2.51), the feedback gain matrix that yields the desired closed-loop eigenvalues and achievable eigenvectors is given by (for independent achievable eigenvectors)

$$G = W[MV_a + NW]^{-1} \quad (2.52)$$

Design Algorithm

A feedback gain matrix that yields a desired closed-loop eigenspace is determined in the following way:

- 1) Select desired eigenvalues λ_{d_i} , desired eigenvectors v_{d_i} , and desired eigenvector weighting matrices Q_{d_i} .
- 2) Calculate w_i 's using equation (2.49) and concatenate these column-wise to form W .
- 3) Calculate achievable eigenvectors v_{a_i} 's using equation (2.47) and concatenate these column-wise to form V_a .
- 4) The feedback gain matrix G is then calculated using equation (2.52).

Existing Lateral-Directional Flying Qualities Criteria and Eigenspace Assignment

A key goal of piloted aircraft control law design is to achieve desirable flying qualities in the closed-loop system. A primary source for flying qualities design criteria for high performance aircraft is the Military Standard 1797A - Flying Qualities of Piloted Aircraft (and earlier versions - Military Standard 1797 and Military Specification 8785). Using eigenspace assignment methods the designer specifies the desired closed-loop dynamics in the form of desired eigenvalues and eigenvectors. The Military Standard provides considerable guidance for choosing lateral-directional eigenvalues to yield desired flying qualities (see Military Standard 1797A sections: 4.5.1.1 - Roll Mode, 4.5.1.2 - Spiral Stability, 4.5.1.3 - Coupled Roll-Spiral Oscillation, 4.6.1.1 - Dynamic Lateral-Directional Response). This guidance is in the form of desired time constants, and frequency and damping specifications.

Unfortunately, the Military Standard provides no direct guidance for choosing lateral-directional eigenvectors to yield desired flying qualities. Indirect guidance is provided in the form of lateral-directional modal coupling requirements. Two sections of Military Standard 1797A directly address lateral-directional coupling for relatively small amplitude rolling maneuvers (see Military Standard 1797A sections: 4.5.1.4 - Roll Oscillations; and 4.6.2 - Yaw Axis Response to Roll Controller). In these sections, requirements are given placing limits on undesirable time responses due to control inputs. These requirements are based on time response parameters that can be measured in flight and were derived from flight data obtained from aircraft possessing conventional modal characteristics. The data base used to define the desired and adequate flying qualities boundaries is drawn from flight test studies conducted during the 60's and 70's. The data used to define this criteria for high performance aircraft is considered to be sparse.

In addition to the Military Standard modal coupling criteria, some guidance is available from Costigan and Calico, 1989. The Costigan-Calico study correlated pilot handling qualities to the ratio of two elements of the Dutch roll eigenvector. Although this study did not lead to a design criteria, it does provide valuable pilot preference information for variations in the studied parameter.

The early lateral-directional flight test studies, the Military Standard requirements for high performance aircraft performing precision tracking tasks, and the Costigan-Calico study are summarized in the following.

Lateral-Directional Flight Test Studies

These studies (Chalk et al. 1969, Chalk et al. 1973) examined the flying qualities for a selected range of lateral-directional dynamics. Different configurations were achieved by varying the system eigenvalues, the roll-to-sideslip ratio, and the bank angle-to-lateral stick transfer function numerator zeros. The roll-to-sideslip ratio $|\phi/\beta|_{dr}$ is defined as the ratio of the amplitudes of the bank angle and sideslip time response envelopes of the dutch roll mode, at any instant in time. Modal characteristics, transfer function zeros, and pilot ratings for a selected set of configurations from one of these studies are given in Tables 1-6.

These studies demonstrated that lateral-directional flying qualities are influenced by the relative location of the bank angle-to-lateral stick (or roll rate-to-lateral stick) transfer function numerator zeros with respect to the dutch roll poles (equation (2.9) or (2.15)). The optimum pilot ratings occurred when the roll rate-to-lateral stick transfer function numerator zeros approximately canceled the dutch roll poles. Configurations with zeros to the left of the dutch roll pole were generally rated better than those with zeros to the right. In addition, configurations with zeros in the lower left quadrant with respect to the dutch roll pole showed less degradation in pilot rating as the zero was moved from its optimum location (See Figure 1).

For configurations with low roll-to-sideslip ratios, the primary concern was the sideslip response that resulted from the lateral stick input rather than the roll response. These configurations have low coupling between the roll and sideslip responses and therefore the roll response is only slightly affected by large sideslip angles. For configurations with medium roll-to-sideslip ratios, the primary concern was the character of the roll response that resulted from the lateral stick input. Configurations with large roll-to-sideslip ratios (along with a lightly damped dutch roll pole) exhibited large rolling moments due to sideslip and were generally found to be unsatisfactory.

As a result of these studies, specifying flying qualities criteria in terms of acceptable roll rate-to-lateral stick transfer function zero locations with respect to the dutch roll pole was investigated. This approach was found to have some major shortcomings. A primary shortcoming was the need to accurately determine the location of the zeros of the roll rate-to-lateral stick transfer function; this is difficult to measure. Industry preferred flying qualities criteria based on easily measured parameters (Chalk et al. 1969). This led to the development of the current time response parameter-based criteria in the Military Standard.

Military Standard Criteria

As shown in the flight test studies, the existence of roll rate oscillations is directly related to the relative locations of the zeros and Dutch roll poles in the roll rate-to-lateral stick transfer function (equation (2.15)). When the complex roots cancel, the Dutch roll mode is not excited at all. When they do not cancel, there is coupling between the roll and sideslip responses. How this coupling is manifested depends upon the magnitude of the roll-to-sideslip ratio for the Dutch roll mode, $|\phi/\beta|_{dr}$. An approximation for the roll-to-sideslip ratio (Chalk et al. 1969) is given by:

$$\left| \frac{\phi}{\beta} \right|_{dr} \cong \left| \frac{L'_\beta}{N'_\beta} \right| \left(\frac{1 + \frac{N'_\beta L_r'^2}{L_\beta'^2}}{1 + \frac{L_p'^2}{N_\beta'^2}} \right)^{1/2} \quad (2.53)$$

The Dutch roll contamination occurs primarily in yaw and sideslip if $|\phi/\beta|_{dr}$ is low (less than approximately 1.5) or primarily in roll rate when $|\phi/\beta|_{dr}$ is moderate-to-large (greater than 3.5 to 5). As $|\phi/\beta|_{dr}$ tends toward zero (L_β tends toward zero), the roll and sideslip responses become less coupled.

In the Military Standard, pilot subjective flying qualities ratings are quantified in terms of Cooper-Harper ratings (Cooper and Harper 1969). The Cooper-Harper rating scale (and its predecessor the Cooper scale (Cooper 1957)), is a numerical scale from one to ten with one being the best rating and ten the worst (see Figure 2). In practice, Cooper-Harper ratings from one through three are referred to as "Level One", ratings from 4 through 6 as "Level Two", and seven through 9 as "Level Three".

Roll Rate Oscillation Criteria

The (p_{osc} / p_{avg}) parameter is directed at precision control of aircraft with moderate-to-high $|\phi/\beta|_{dr}$ combined with marginally low Dutch roll damping. The ratio (p_{osc} / p_{avg}) is a measure of the ratio of the oscillatory component of the roll rate to the average component of the roll rate following a step roll command (Chalk et al. 1969). This ratio is defined as

$$\frac{p_{osc}}{p_{avg}} = \frac{p_1 + p_3 - 2p_2}{p_1 + p_3 + 2p_2} \quad (2.54)$$

for ζ_{dr} less than or equal to 0.2 and

$$\frac{p_{osc}}{p_{avg}} = \frac{p_1 - p_2}{p_1 + p_2} \quad (2.55)$$

for ζ_{dr} greater than 0.2 where p_1 , p_2 , and p_3 are roll rates at the first, second, and third peaks; respectively.

The values of (p_{osc} / p_{avg}) that a pilot will accept are a function of the angular position of the zero relative to the Dutch roll pole in the roll rate-to-lateral stick transfer function. This angle will be referred to as Ψ_1 . Values of Ψ_1 for various zero locations are given in Figure 3. Because of the difficulty in directly measuring Ψ_1 , the criteria is specified in

terms of the phase angle of the Dutch roll oscillation in sideslip (for a step input), Ψ_β (see equation (2.37)). This angle can be measured from the sideslip time response due to a step input. The angle Ψ_1 is directly related to the angle, Ψ_β . For positive dihedral, this relationship is given by (Chalk et al. 1969)

$$\Psi_\beta \cong \Psi_1 - 270 \text{ (degrees)} \quad (2.56)$$

This relationship is relatively independent of roll and spiral eigenvalue locations and holds for a wide range of stability derivatives.

The (p_{osc} / p_{avg}) criteria (for positive dihedral) is given in Figure 4. The Level One flying qualities boundary has a constant magnitude of 0.05 for $0 \geq \Psi_\beta \geq -130$ and $-340 \geq \Psi_\beta \geq -360$ degrees and a constant magnitude of 0.25 for $-200 \geq \Psi_\beta \geq -270$ degrees. The magnitude increases linearly from a magnitude of 0.05 at $\Psi_\beta = -130$ degrees to a magnitude of 0.25 at $\Psi_\beta = -200$ degrees. The magnitude decreases linearly from a magnitude of 0.25 at $\Psi_\beta = -270$ degrees to a magnitude of 0.05 at $\Psi_\beta = -340$ degrees.

For all flying qualities levels, the change in bank angle must always be in the direction of the lateral stick control command. This requirement applies for step roll commands up to the magnitude which causes a 60 degree bank angle change in $1.7 T_d$ seconds, where T_d is the damped period of the Dutch roll eigenvalue.

$$T_d = \frac{2\pi}{\omega_{dr} \sqrt{1 - \zeta_{dr}^2}} \quad (2.57)$$

The primary source of data upon which this (p_{osc} / p_{avg}) requirement is based is the medium $|\phi/\beta|_{dr}$ configurations of Meeker and Hall, 1967.

Sideslip Excursion Criteria

The ($\Delta\beta_{max} / k_\beta$) parameter applies to sideslip excursions and is directed at aircraft with low-to-moderate $|\phi/\beta|_{dr}$. The term $\Delta\beta_{max}$ is defined as the maximum sideslip excursion at the c.g. for a step roll command

$$\Delta\beta_{max} = \max(\beta(t)) - \min(\beta(t)) \quad \text{for } 0 < t < t_\beta \quad (2.58)$$

where t_β is equal to 2 seconds or one half period of the Dutch roll, whichever is greater. The term k_β is defined as the ratio of “achieved roll performance” to “roll performance requirement”

$$k_\beta = \frac{\phi(t)}{\phi_{req}} \bigg|_{t=t_{req}} \quad (2.59)$$

where $\phi(t)$ is the bank angle at a specified period of time, t_{req} and ϕ_{req} is the bank angle requirement specified in the Military Standard (Chalk et al. 1969). For example, a ϕ_{req} typically used for high performance aircraft is 60 degrees bank angle at one second. For this requirement, equation (2.59) reduces to

$$k_\beta = \frac{\phi(t)}{60} \bigg|_{t=1 \text{ sec}} \quad (2.60)$$

where $\phi(t)$ has units of degrees.

The amount of sideslip that a pilot will tolerate is a function of the phase angle of the Dutch roll component of sideslip, Ψ_β (equation (2.56)). When the phase angle is such that β is primarily adverse (out of the turn being rolled into), the pilot can tolerate a considerable amount of sideslip. When the phasing is such that β is primarily proverse (into the turn), the pilot can only tolerate a small amount of sideslip because of the difficulty of coordination.

The $(\Delta\beta_{max} / k\beta)$ requirement is given in Figure 5. As shown, the Level One flying qualities boundary has a constant magnitude of 2 for $0 \geq \Psi_\beta \geq -130$ and $-340 \geq \Psi_\beta \geq -360$ degrees and a constant magnitude of 6 for $-200 \geq \Psi_\beta \geq -270$ degrees. The magnitude increases linearly from a magnitude of 2 at $\Psi_\beta = -130$ degrees to a magnitude of 6 at $\Psi_\beta = -200$ degrees. The magnitude decreases linearly from a magnitude of 6 at $\Psi_\beta = -270$ degrees to a magnitude of 2 at $\Psi_\beta = -340$ degrees.

This requirement applies for step roll control commands up to the magnitude that causes a 60 degree bank angle change within T_d or 2 seconds, whichever is longer. The primary source of data from which the sideslip requirement for high performance aircraft evolved is the low $|\phi/\beta|_{dr}$ (approximately 1.5) configurations of Meeker and Hall, 1967. In general the available data suggest that $(\Delta\beta_{max} / k\beta)$ is not as useful as (p_{osc} / p_{avg}) when $|\phi/\beta|_{dr} > 3.5$ to 5.0.

Costigan-Calico Flight Test Study

The primary objective of this study (Costigan and Calico 1989) was to correlate pilot handling qualities to the magnitude and phase of the roll-to-sideslip ratio for the Dutch roll mode, $|\phi/\beta|_{dr}$. Seven combinations of roll-to-sideslip ratio magnitude ($|\phi/\beta|_{dr}$) (0, 1.5, and, 3.0) and phase angle ($\angle(\phi/\beta)_{dr}$) (0, 60, and 120 degrees) were tested (Table 7). These variations were achieved by varying the magnitude and phase of the ratio of the ϕ and β elements of the Dutch roll eigenvector. System eigenvalues, and roll and spiral eigenvectors were set to desired values and not varied (Table 8). Control laws were designed using eigenspace assignment and flight tested on a YA-7D test aircraft. Three pilots evaluated each of these seven configurations using two Heads-Up-Display tracking tasks (yaw pointing and bank angle tracking) and an air-to-air task with a cooperative target. Pilot ratings were given in terms of Cooper-Harper ratings (Cooper and Harper 1969). Average Cooper-Harper ratings for the yaw pointing and bank angle tracking tasks are summarized in Table 9.

Overall, the results showed little variation of the pilot ratings with $|\phi/\beta|_{dr}$ and a preference for zero degree roll-to-sideslip phase angle over the larger phase angles. Costigan and Calico state that they believed the poor lateral stick dynamics of the YA-7D test aircraft degraded the lateral flying qualities ratings in all tasks and contributed to the small variations in pilot rating with $|\phi/\beta|_{dr}$.

As shown, the Military Standard provides indirect guidance for choosing lateral-directional eigenvectors to yield desired flying qualities in the form of the roll rate oscillation and the sideslip excursion criteria. Using these criteria, it is not clear how to choose eigenvectors to achieve desired closed-loop flying qualities, or trade-off flying qualities for other important design requirements, such as achieving realizable gain magnitudes or desired system robustness. The next section addresses this shortcoming by presenting the development of guidelines for choosing lateral-directional eigenvectors to

yield desired flying qualities. This is done by developing relationships between the lateral-directional eigenvectors and the roll rate and sideslip transfer functions. Using these relationships, along with constraints imposed by system dynamics, key eigenvector elements are identified and guidelines for choosing values of these elements to yield desirable flying qualities developed.

3.0 LATERAL-DIRECTIONAL EIGENVECTOR DESIGN GUIDELINES

Eigenspace assignment methods allow designers to shape the closed-loop response by judicious choice of desired eigenvalues and eigenvectors. As shown earlier, the eigenvalues determine the time constant or the frequency and damping of each mode and the eigenvectors determine how much each mode of the system contributes to each output.

When choosing desired closed-loop eigenvectors, the designer is faced with two challenges. First, the designer must choose which of the elements of the eigenvector matrix to specify. Using eigenspace assignment methods, for a system with n states, m controls, and l measurements, one has the freedom to place l eigenvalues to desired locations and m elements of their associated eigenvectors to desired values. Since for aircraft there are usually fewer controls than states, only a subset of the system eigenvectors can be exactly specified. Secondly, once the designer has chosen which elements to specify, he/she must decide what values to specify. Currently, no guidelines exist for choosing lateral-directional eigenvector elements to yield desirable flying qualities. Design guidelines would allow designers to perform trade-offs between flying qualities and other important design requirements, such as achieving realizable gain magnitudes or desired system robustness.

This section addresses these two challenges by developing relationships between the system's eigenvectors and the roll rate and sideslip transfer functions. Using these relationships, along with constraints imposed by system dynamics, key eigenvector elements are identified and flying qualities guidelines for choosing appropriate values of these elements developed.

Transfer Functions and Eigenvector Element Ratios

Because eigenvectors can be scaled by an arbitrary constant, individual elements of an eigenvector are not unique. But, ratios of two elements of the same eigenvector are unique. Because of this, the eigenvector relationships and guidelines developed in this section will be stated in terms of these ratios. These ratios will be referred to as eigenvector element ratios. An eigenvector element ratio is equal to the ratio of the x_i and x_j elements in the eigenvector associated with mode k and will be denoted by

$$\left(\frac{x_i}{x_j} \right)_{\text{mode } k} \quad (3.1)$$

Eigenvector element ratios (also referred to as modal response ratios) can be expressed using any one of the n cofactors of the system's characteristic determinant (McRuer et al. 1973). The eigenvector element ratio between two states x_i and x_j , and evaluated at mode k , is given by

$$\left(\frac{x_i}{x_j} \right)_{\text{mode } k} = (-1)^{(i-j)} \left(\frac{\Delta_{qi}(s)}{\Delta_{qj}(s)} \right) \Big|_{s=\lambda_k} \quad \text{for } q = 1, \dots, n \quad (3.2)$$

where $\Delta_{qi}(s)$ and $\Delta_{qj}(s)$ are the minors of the characteristic determinant $\Delta(s)$ of the system given by equation (2.1).

Roll Rate Oscillations and Eigenvector Element Ratios

The existence of roll rate oscillations is directly traceable to the relative locations of the zeros and Dutch roll poles in the p-to- δ_{stk} transfer function (equation (2.15)). Equations for the frequency and real part of the Dutch roll pole are given by equations (2.4) and (2.5), respectively. Equations for the p-to- δ_{stk} zeros, as a function of the Dutch roll frequency and damping and lateral-directional coupling derivatives are determined as follows.

Substituting equation (2.4) into equation (2.10) and making the assumptions of (2.12) yields

$$\omega_\phi^2 \equiv \omega_{dr}^2 \left(1 - \left(\frac{N'_\delta}{L'_\delta} \right) \left(\frac{L'_\beta}{N'_\beta} \right) \right) \quad (3.3)$$

Subtracting equation (2.5) from equation (2.11) yields

$$2\omega_\phi \zeta_\phi \equiv 2\omega_{dr} \zeta_{dr} + \left(\frac{N'_\delta}{L'_\delta} \right) L'_r + \left(\frac{L'_\beta}{N'_\beta} \right) \left(N'_p - \frac{g}{V_0} \right) \quad (3.4)$$

As shown by these equations, the location of the zeros of the p-to- δ_{stk} transfer function with respect to the Dutch roll poles are primarily a function of the control coupling derivatives (N'_δ/L'_δ); and the stability coupling derivatives (L'_β/N'_β), ($N'_p - g/V_0$), and L'_r .

The control derivatives are elements of the control matrix (B matrix of equation (2.22)) and are therefore independent of system eigenvectors. The ratio of control derivatives (N'_δ/L'_δ) can be adjusted by blending the lateral and directional control effectors (e.g. an aileron-rudder interconnect). The stability derivatives are elements of the state matrix (A matrix of equation (2.22)) and can be related to the system eigenvectors. This is done in the following.

Solution for L'_r

The eigenvector element ratio (ϕ/β) is given by applying equation (3.2) with $i=2$ and $j=1$

$$\left(\frac{\phi}{\beta} \right) = - \left(\frac{\Delta_{32}}{\Delta_{31}} \right) = \frac{-L'_r(s - Y_\beta) + L'_\beta}{s^2 - L'_p s - \frac{g}{V_0} L'_r} \quad (3.5)$$

where $q=3$ is chosen to yield a desired solution form. Evaluating this ratio at $s = \lambda_{dr}$, and recognizing that both $Y_\beta L'_r \ll L'_\beta$ and $(g/V_0)L'_r \ll \omega_{dr}^2$ yields

$$\left(\frac{\phi}{\beta}\right)_{dr} \equiv \frac{-\dot{L}_r \lambda_{dr} + \dot{L}_\beta}{\lambda_{dr}^2 - \dot{L}_p \lambda_{dr}} \quad (3.6)$$

The term $(\phi/\beta)_{dr}$ is the ratio of the bank angle and sideslip elements in the Dutch roll eigenvector. The phase angle of equation (3.6) is given by

$$\begin{aligned} \angle\left(\frac{\phi}{\beta}\right)_{dr} &\equiv -\text{atan}\left(\frac{\dot{L}_r \omega_{dr} \sqrt{1-\zeta_{dr}^2}}{\dot{L}_\beta + \zeta_{dr} \omega_{dr} \dot{L}_r}\right) + \text{atan}\left(\frac{\omega_{dr} \sqrt{1-\zeta_{dr}^2} (\dot{L}_p + 2\zeta_{dr} \omega_{dr})}{\omega_{dr}^2 (2\zeta_{dr}^2 - 1) + \zeta_{dr} \omega_{dr} \dot{L}_p}\right) \\ &= \text{atan}\left(\frac{\omega_{dr} \sqrt{1-\zeta_{dr}^2} (\dot{L}_p \dot{L}_\beta + 2\zeta_{dr} \omega_{dr} \dot{L}_\beta + \omega_{dr}^2 \dot{L}_r)}{\omega_{dr}^2 (2\zeta_{dr}^2 - 1) \dot{L}_\beta + \zeta_{dr} \omega_{dr}^3 \dot{L}_r + \zeta_{dr} \omega_{dr} \dot{L}_p \dot{L}_\beta + \omega_{dr}^2 \dot{L}_r \dot{L}_p}\right) \end{aligned} \quad (3.7)$$

Solving this equation for \dot{L}_r yields

$$\dot{L}_r \equiv -\left(\frac{\dot{L}_\beta}{\dot{N}_\beta}\right) \left[\frac{(\dot{L}_p + 2\zeta_{dr} \omega_{dr}) \omega_{dr} \sqrt{1-\zeta_{dr}^2} - (\omega_{dr}^2 (2\zeta_{dr}^2 - 1) + \zeta_{dr} \omega_{dr} \dot{L}_p) \tan \angle\left(\frac{\phi}{\beta}\right)_{dr}}{\omega_{dr} \sqrt{1-\zeta_{dr}^2} - (\dot{L}_p + \zeta_{dr} \omega_{dr}) \tan \angle\left(\frac{\phi}{\beta}\right)_{dr}} \right] \quad (3.8)$$

This phase angle $\angle(\phi/\beta)_{dr}$ is related to the phase angle $\angle(p/\beta)_{dr}$ by the following relation

$$\angle\left(\frac{\phi}{\beta}\right)_{dr} = \angle\left(\frac{p}{\beta}\right)_{dr} - \angle \lambda_{dr} \quad (3.9)$$

The phase angle $\angle(p/\beta)_{dr}$ is a discriminator of positive and negative dihedral (Chalk et al. 1969). Positive dihedral corresponds to $45^\circ < \angle(p/\beta)_{dr} < 225^\circ$. For a stable complex Dutch roll pole

$$\angle \lambda_{dr} = 180^\circ - a \cos(\zeta_{dr}) \quad (3.10)$$

Therefore, for positive dihedral (and a stable complex Dutch roll pole)

$$-135^\circ + a \cos(\zeta_{dr}) < \angle\left(\frac{\phi}{\beta}\right)_{dr} < 45^\circ + a \cos(\zeta_{dr}) \quad (3.11)$$

where $0^\circ < a \cos(\zeta_{dr}) < 90^\circ$. Results from Costigan and Calico (1989) show a pilot preference for $\angle(\phi/\beta)_{dr} = 0$. Choosing $\angle(\phi/\beta)_{dr} = 0$ provides positive dihedral for any stable complex Dutch roll pole and simplifies the solution for \dot{L}_r . For $\angle(\phi/\beta)_{dr} = 0$, equation (3.8) reduces to

$$\dot{L}_r \cong -\left(\frac{\dot{L}_\beta}{\dot{N}_\beta}\right)\left(\dot{L}_p + 2\zeta_{dr}\omega_{dr}\right) \quad (3.12)$$

A relationship between λ_{roll} , L'_p , and coupling derivatives is given by equation (2.6). Solving this for L'_p yields

$$\dot{L}_p \cong \lambda_{roll} + \left(\frac{\dot{L}_\beta}{\dot{N}_\beta}\right)\left(\dot{N}_p - \frac{g}{V_0}\right) \quad (3.13)$$

Substituting this equation for L'_p (equation (3.13)) into (3.12) yields

$$\dot{L}_r \cong -\left(\frac{\dot{L}_\beta}{\dot{N}_\beta}\right)\left(\lambda_{roll} + \left(\frac{\dot{L}_\beta}{\dot{N}_\beta}\right)\left(\dot{N}_p - \frac{g}{V_0}\right) + 2\zeta_{dr}\omega_{dr}\right) \quad (3.14)$$

Solution for $(N'_p - g/V_0)$

The eigenvector element ratio (β/p) is given by applying equation (3.2)

$$\left(\frac{\beta}{p}\right) = -\left(\frac{\Delta_{31}}{\Delta_{32}}\right) = \frac{-\left(\frac{g}{V_0 s}\right)\dot{L}_r + (s - \dot{L}_p)}{\dot{L}_\beta - \dot{L}_r(s - Y_\beta)} \quad (3.15)$$

Evaluating this ratio at $s = \lambda_{roll}$ and recognizing that both $Y_\beta L'_r \ll L'_\beta$ and $(g/V_0)L'_r/\lambda_{roll} \ll L'_p$ yields

$$\left(\frac{\beta}{p}\right)_{roll} \cong \frac{\lambda_{roll} - \dot{L}_p}{\dot{L}_\beta - \lambda_{roll}\dot{L}_r} \quad (3.16)$$

The term $(\beta/p)_{roll}$ is the ratio of the sideslip and roll rate elements in the roll mode eigenvector. Substituting equation (3.13) for L'_p into (3.16) yields

$$\left(\frac{\beta}{p}\right)_{roll} \cong \frac{-\left(\frac{\dot{L}_\beta}{\dot{N}_\beta}\right)\left(\dot{N}_p - \frac{g}{V_0}\right)}{\dot{L}_\beta - \lambda_{roll}\dot{L}_r} \quad (3.17)$$

Substituting equation (3.14) for L'_r into (3.17) and applying $\omega_{dr}^2 \cong \dot{N}_\beta$ yields

$$\left(\frac{\beta}{p}\right)_{roll} \cong \frac{-\left(\dot{N}_p - \frac{g}{V_0}\right)}{\omega_{dr}^2 + \lambda_{roll}\left(\lambda_{roll} + 2\zeta_{dr}\omega_{dr} + \left(\frac{\dot{L}_\beta}{\dot{N}_\beta}\right)\left(\dot{N}_p - \frac{g}{V_0}\right)\right)} \quad (3.18)$$

Solving for $(N'_p - g/V_0)$ yields

$$\left(N'_p - \frac{g}{V_0}\right) \cong \frac{-\left(\frac{\beta}{p}\right)_{roll} \left(\lambda_{roll}^2 + 2\lambda_{roll}\zeta_{dr}\omega_{dr} + \omega_{dr}^2\right)}{\left(1 + \lambda_{roll}\left(\frac{\beta}{p}\right)_{roll}\left(\frac{\dot{L}_\beta}{\dot{N}_\beta}\right)\right)} \quad (3.19)$$

Solution for (L'_β/N'_β)

Substituting equation (3.14) for L'_r into (3.6) and applying $\omega_{dr}^2 \cong N'_\beta$ yields

$$\left(\frac{\phi}{\beta}\right)_{dr} \cong \left(\frac{\dot{L}_\beta}{\dot{N}_\beta}\right) \left[\frac{\left(\dot{L}_p + 2\zeta_{dr}\omega_{dr}\right)\lambda_{dr} + \omega_{dr}^2}{\lambda_{dr}^2 - \dot{L}_p\lambda_{dr}} \right] \quad (3.20)$$

By noting that the term in square brackets is equal to -1 (and that for $\angle(\phi/\beta)_{dr}=0$ the ratio $(\phi/\beta)_{dr}$ is a real number) equation (3.20) reduces to (for positive dihedral)

$$\left(\frac{\phi}{\beta}\right)_{dr} = \left|\frac{\phi}{\beta}\right|_{dr} \cong -\left(\frac{\dot{L}_\beta}{\dot{N}_\beta}\right) \quad (3.21)$$

By substituting equations (3.14), (3.19), and (3.21) into (3.3) and (3.4); one can now obtain expressions for the frequency and damping of the transfer function zeros as a function of ω_{dr} , ζ_{dr} , λ_{roll} , (N'_δ/L'_δ) , and eigenvector element ratios. This is done in the following steps.

Substituting equation (3.21) into (3.3) yields

$$\omega_\phi^2 \cong \omega_{dr}^2 \left(1 + \left(\frac{N'_\delta}{L'_\delta}\right) \left|\frac{\phi}{\beta}\right|_{dr} \right) \quad (3.22)$$

Substituting equation (3.14) for L'_r into (3.4) yields

$$\begin{aligned} 2\omega_\phi\zeta_\phi \cong & 2\omega_{dr}\zeta_{dr} - \left(\frac{\dot{L}_\beta}{\dot{N}_\beta}\right) \left(\frac{N'_\delta}{L'_\delta}\right) \left(\lambda_{roll} + \left(\frac{\dot{L}_\beta}{\dot{N}_\beta}\right) \left(N'_p - \frac{g}{V_0}\right) + 2\zeta_{dr}\omega_{dr} \right) \\ & + \left(\frac{\dot{L}_\beta}{\dot{N}_\beta}\right) \left(N'_p - \frac{g}{V_0}\right) \end{aligned} \quad (3.23)$$

Rearranging terms yields

$$2\omega_\phi \zeta_\phi \cong 2\omega_{dr} \zeta_{dr} - \left(\frac{\dot{L}_\beta}{N_\beta} \right) \left\{ \left(\frac{N'_\delta}{\dot{L}_\delta} \right) (\lambda_{roll} + 2\zeta_{dr} \omega_{dr}) - \left(N_p - \frac{g}{V_0} \right) \left(1 - \left(\frac{\dot{L}_\beta}{N_\beta} \right) \left(\frac{N'_\delta}{\dot{L}_\delta} \right) \right) \right\} \quad (3.24)$$

Now substituting equation (3.19) for $(N'_p - g/V_0)$ into (3.24) yields

$$2\omega_\phi \zeta_\phi \cong 2\omega_{dr} \zeta_{dr} - \left(\frac{\dot{L}_\beta}{N_\beta} \right) \left\{ \left(\frac{N'_\delta}{\dot{L}_\delta} \right) (\lambda_{roll} + 2\zeta_{dr} \omega_{dr}) + \left(\frac{\beta}{p} \right)_{roll} \frac{(\lambda_{roll}^2 + 2\lambda_{roll} \zeta_{dr} \omega_{dr} + \omega_{dr}^2)}{\left(1 + \lambda_{roll} \left(\frac{\beta}{p} \right)_{roll} \left(\frac{\dot{L}_\beta}{N_\beta} \right) \right)} \left(1 - \left(\frac{\dot{L}_\beta}{N_\beta} \right) \left(\frac{N'_\delta}{\dot{L}_\delta} \right) \right) \right\} \quad (3.25)$$

Finally, substituting equation (3.21) for (L'_β / N'_β) into equation (3.25) yields the desired relationship

$$2\omega_\phi \zeta_\phi \cong 2\omega_{dr} \zeta_{dr} + \left| \frac{\phi}{\beta} \right|_{dr} \left\{ \left(\frac{N'_\delta}{\dot{L}_\delta} \right) (\lambda_{roll} + 2\zeta_{dr} \omega_{dr}) + \left(\frac{\beta}{p} \right)_{roll} \frac{(\lambda_{roll}^2 + 2\lambda_{roll} \zeta_{dr} \omega_{dr} + \omega_{dr}^2)}{\left(1 - \lambda_{roll} \left(\frac{\beta}{p} \right)_{roll} \left| \frac{\phi}{\beta} \right|_{dr} \right)} \left(1 + \left| \frac{\phi}{\beta} \right|_{dr} \left(\frac{N'_\delta}{\dot{L}_\delta} \right) \right) \right\} \quad (3.26)$$

Equations (3.22) and (3.26) provide equations for ω_ϕ and ζ_ϕ as a function of system eigenvalues, eigenvector element ratios ($|\phi/\beta|_{dr}$ and $(\beta/p)_{roll}$) and the adverse yaw ratio (N'_δ / L'_δ). The relationship between $(\beta/p)_{roll}$ and (N'_δ / L'_δ) and ω_ϕ and ζ_ϕ is demonstrated in Figure 6 for $|\phi/\beta|_{dr} = 5$ and eigenvalues $\lambda_{roll} = -2.5$, $\lambda_{dr} = (\omega_{dr} = 2.0$ (rad/sec), $\zeta_{dr} = 0.1$). In this figure, lines of constant ω_ϕ are given by solid lines (1.5, 2, 2.5, 3 (rad/sec)) and lines of constant ζ_ϕ are given by dashed lines (0, 0.4, 0.7, 1).

As can be seen from equation (3.26), the value of the ratio $|\phi/\beta|_{dr}$ has a strong effect on the transfer function zero locations. For $|\phi/\beta|_{dr} = 0$, the locations of the transfer function zeros are not affected by variations in $(\beta/p)_{roll}$ and (N'_δ/L'_δ) . As $|\phi/\beta|_{dr}$ increases, the zeros become more sensitive to variations in $(\beta/p)_{roll}$ and (N'_δ/L'_δ) .

Sideslip Response and Eigenvector Element Ratios

The sideslip-to-lateral stick transfer function can be written as (equation (2.19))

$$\frac{\beta}{\delta_{stk}} = \frac{-N'_\delta \left(s + \left(-L'_p + \frac{L'_\delta}{N'_\delta} \left(N'_p - \frac{g}{V_0} \right) \right) \right)}{(s - \lambda_{roll})(s^2 + 2\zeta_{dr}\omega_{dr}s + \omega_{dr}^2)} \quad (3.27)$$

Rearranging terms in the numerator yields

$$\frac{\beta}{\delta_{stk}} = \frac{-L'_\delta \left(\frac{N'_\delta}{L'_\delta} (s - L'_p) + \left(N'_p - \frac{g}{V_0} \right) \right)}{(s - \lambda_{roll})(s^2 + 2\zeta_{dr}\omega_{dr}s + \omega_{dr}^2)} \quad (3.28)$$

By substituting equations (3.13), (3.19), and (3.21) into (3.28); one can now obtain an expression for the sideslip-to-lateral stick transfer function zero as a function of system eigenvalues, eigenvector element ratios, and adverse yaw ratio. This is done in the following steps.

Substituting equation (3.13) for L'_p into equation (3.28) yields

$$\frac{\beta}{\delta_{stk}} = \frac{-L'_\delta \left(\frac{N'_\delta}{L'_\delta} (s - \lambda_{roll}) + \left(N'_p - \frac{g}{V_0} \right) \left(1 - \frac{N'_\delta}{L'_\delta} \left(\frac{L'_\beta}{N'_\beta} \right) \right) \right)}{(s - \lambda_{roll})(s^2 + 2\zeta_{dr}\omega_{dr}s + \omega_{dr}^2)} \quad (3.29)$$

Now substituting equation (3.19) for $(N'_p - g/V_0)$ into (3.29) yields

$$\frac{\beta}{\delta_{stk}} = \frac{-L'_\delta \left(\frac{N'_\delta}{L'_\delta} (s - \lambda_{roll}) - \left(1 - \frac{N'_\delta}{L'_\delta} \left(\frac{L'_\beta}{N'_\beta} \right) \right) \left(\frac{\beta}{p} \right)_{roll} \frac{(\lambda_{roll}^2 + 2\lambda_{roll}\zeta_{dr}\omega_{dr} + \omega_{dr}^2)}{\left(1 + \lambda_{roll} \left(\frac{\beta}{p} \right)_{roll} \left(\frac{L'_\beta}{N'_\beta} \right) \right)} \right)}{(s - \lambda_{roll})(s^2 + 2\zeta_{dr}\omega_{dr}s + \omega_{dr}^2)} \quad (3.30)$$

Finally, substituting equation (3.21) for $(L'\beta / N'\beta)$ into equation (3.30) yields the desired relationship

$$\frac{\beta}{\delta_{stk}} \equiv \frac{-\dot{L}'_{\delta} \left[\frac{N'_{\delta}}{\dot{L}'_{\delta}} (s - \lambda_{roll}) - \left(1 + \frac{N'_{\delta}}{\dot{L}'_{\delta}} \left| \frac{\phi}{\beta} \right|_{dr} \right) \frac{\left(\frac{\beta}{p} \right)_{roll} (\lambda_{roll}^2 + 2\lambda_{roll}\zeta_{dr}\omega_{dr} + \omega_{dr}^2)}{\left(1 - \lambda_{roll} \left(\frac{\beta}{p} \right)_{roll} \left| \frac{\phi}{\beta} \right|_{dr} \right)} \right]}{(s - \lambda_{roll})(s^2 + 2\zeta_{dr}\omega_{dr}s + \omega_{dr}^2)} \quad (3.31)$$

Equation (3.31) provides an equation for the sideslip response as a function of system eigenvalues, eigenvector element ratios ($|\phi/\beta|_{dr}$ and $(\beta/p)_{roll}$) and the adverse yaw ratio (N'_{δ}/L'_{δ}).

Additional Eigenvector Element Ratio Relationships

This section presents additional eigenvector element ratio relationships imposed by the system dynamics.

Roll Rate-to-Bank Angle Ratios

The roll rate-to-bank angle ratios for each mode can be determined from the kinematic relationship between roll rate and bank angle

$$\frac{p}{\delta_{stk}} = \frac{s\phi}{\delta_{stk}} \quad (3.32)$$

Evaluating this equation at each mode yields the following relationships

$$\left(\frac{p}{\phi} \right)_{dr} = \lambda_{dr}; \quad \left(\frac{p}{\phi} \right)_{roll} = \lambda_{roll}; \quad \left(\frac{p}{\phi} \right)_{sprl} = \lambda_{sprl} \quad (3.33)$$

Yaw Rate-to-Sideslip Ratio in the Dutch Roll Mode

The yaw-to-sideslip ratio in the Dutch roll mode can be determined from the sideforce equation of (2.1). This is given by (making the assumptions of (2.12))

$$\frac{r}{\beta} - \frac{g}{V_0} \left(\frac{\phi}{\beta} \right) + (s - Y_{\beta}) = 0 \quad (3.34)$$

Evaluating this equation at $s = \lambda_{dr}$, and solving for $(r/\beta)_{dr}$ yields

$$\left(\frac{r}{\beta} \right)_{dr} = -\lambda_{dr} + Y_{\beta} + \frac{g}{V_0} \left(\frac{\phi}{\beta} \right)_{dr} \quad (3.35)$$

In general, both Y_β and $(g/V_0)(\phi/\beta)_{dr}$ are very small compared to λ_{dr} , therefore

$$\left(\frac{r}{\beta}\right)_{dr} \cong -\lambda_{dr} \quad (3.36)$$

Yaw Rate-to-Roll Rate in Roll Mode

The yaw-to-roll rate ratio in the roll mode can also be determined from the sideforce equation of (3.34). Evaluating this equation at $s=\lambda_{roll}$, and solving for $(r/\beta)_{roll}$ yields

$$\left(\frac{r}{\beta}\right)_{roll} = -(\lambda_{roll} - Y_\beta) + \frac{g}{V_0} \left(\frac{\phi}{\beta}\right)_{roll} \quad (3.37)$$

Multiplying both sides by $(\beta/p)_{roll}$ and applying equation (3.33) yields

$$\left(\frac{r}{p}\right)_{roll} = -\left(\frac{\beta}{p}\right)_{roll} (\lambda_{roll} - Y_\beta) + \frac{g}{V_0 \lambda_{roll}} \quad (3.38)$$

In general, Y_β is very small compared to λ_{roll} , therefore

$$\left(\frac{r}{p}\right)_{roll} \cong -\left(\frac{\beta}{p}\right)_{roll} \lambda_{roll} + \frac{g}{V_0 \lambda_{roll}} \quad (3.39)$$

Yaw Rate-to-Bank Angle Ratio in the Spiral Mode

The yaw-to-bank angle ratio in the spiral mode can be determined from the sideforce equation of (3.34). Evaluating this equation at $s=\lambda_{sprl}$, and solving for $(r/\beta)_{sprl}$ yields

$$\left(\frac{r}{\beta}\right)_{sprl} = -\lambda_{sprl} + Y_\beta + \frac{g}{V_0} \left(\frac{\phi}{\beta}\right)_{sprl} \quad (3.40)$$

Multiplying both sides by $(\beta/\phi)_{sprl}$ and assuming $(\beta/\phi)_{sprl} (Y_\beta - \lambda_{sprl})$ is small compared to (g/V_0) yields

$$\left(\frac{r}{\phi}\right)_{sprl} \cong \frac{g}{V_0} \quad (3.41)$$

Specifying the Desired Eigenvectors in terms of Eigenvector Element Ratios

Desired eigenvector elements will be chosen by considering both the relationships between the eigenvector element ratios and transfer functions, and constraints imposed by system dynamics. Desired eigenvector elements will be specified for the lateral-directional state vector given by

$$\underline{x} = \{ \beta \ p \ r \ \phi \}^T \quad (3.42)$$

For lateral-directional design, usually only two controls are available - roll moment and yaw moment (the direct sideforce generated by conventional controls is usually small). This development will assume that four measurements (or full-state feedback) are available. Therefore, using eigenspace assignment methods one can exactly place all four lateral-directional eigenvalues and only two elements of each associated eigenvector.

Of course, more than two elements in each desired eigenvector can be specified. This results in closed-loop eigenvectors that are as close as possible in a least squares sense to the desired eigenvectors. In this authors experience, for fixed desired eigenvalues, the resulting closed-loop eigenvectors are usually a poor fit to the desired eigenvectors. (Although not considered in this work, eigenvector fit can be improved by allowing variations in the desired eigenvalues within a certain region). Therefore, this work will only consider specifying two elements of each eigenvector.

Because eigenvectors can be scaled by an arbitrary constant, one element of each eigenvector will be specified to be unity to ensure that the eigenvectors are unique. In the following desired eigenvectors, a * indicates that the element is not specified and therefore not weighted in the cost function.

Spiral Eigenvector

The spiral mode is a first order mode with a long time constant. Classically, the spiral mode is dominant in bank angle and almost nonexistent in sideslip. Therefore, the bank angle element will be chosen to be unity. The eigenvector element ratios available to be specified for this eigenvector are (β/ϕ) , (p/ϕ) , and (r/ϕ) . Having a very small spiral mode contribution to sideslip is desirable because it results in coordinated banking and turning. This can be achieved by choosing the (β/ϕ) to be zero. This results in $R_{\beta, sprl} = 0$. The two remaining ratios (r/ϕ) and (p/ϕ) are constrained by the system dynamics. The ratio of the p and ϕ elements is constrained to be equal to the spiral eigenvalue (equation (3.33)) and the ratio of the r and ϕ elements is constrained to be approximately equal to (g/V_0) (equation (3.41)). Therefore, the desired spiral mode eigenvector is specified to be

$$\underline{v}_{sprl} = [0 \quad * \quad * \quad 1]^T \quad (3.43)$$

Roll Eigenvector

The roll mode is a first order mode with a relatively short time constant. Classically, the roll mode is dominant in roll rate therefore this element will be chosen to be unity. Therefore, the eigenvector element ratios available to be specified for this eigenvector are (β/p) , (r/p) , and (ϕ/p) . As was shown by equation (3.25), the ratio of the β and p elements, $(\beta/p)_{roll}$, effects the cancellation of the Dutch roll mode in the roll response. The two remaining ratios, (r/p) and (ϕ/p) , are constrained by the system dynamics. The ratio of the ϕ and p elements is equal to the inverse of the roll eigenvalue (equation (3.33)) and the ratio of the r and p elements is a function of the roll eigenvalue and (β/p) (equation (3.39)). Therefore, the roll mode eigenvector is specified to be

$$\underline{v}_{roll} = \left[\left(\frac{\beta}{p} \right)_{roll} \quad 1 \quad * \quad * \right]^T \quad (3.44)$$

Dutch Roll Eigenvector

The Dutch roll mode is a lightly damped oscillatory mode. For this eigenvector, the sideslip element will be chosen to be unit magnitude with zero phase. Therefore, the eigenvector element ratios available to be specified for this eigenvector are (p/β) , (r/β) , and (ϕ/β) . Specifying the ratio of the β and ϕ elements in the Dutch roll eigenvector determines the $|\phi/\beta|_{dr}$ ratio. Results from Costigan and Calico (1989) show a pilot preference for $\angle(\phi/\beta)_{dr} = 0$. The two remaining ratios, (p/β) and (r/β) are constrained by the system dynamics. The ratio of the r and β elements is constrained by the system dynamics to be approximately equal to the negative of the Dutch roll eigenvalue (equation (3.36)). The ratio (p/β) can be written as the product of the (ϕ/β) and (p/ϕ) ratios. Because the ratio of the p and ϕ elements is constrained to be equal to the Dutch roll eigenvalue (equation (3.33)), once the (ϕ/β) ratio is specified then the (p/β) ratio is also specified. Therefore, the Dutch roll mode eigenvector is specified to be

$$\underline{v}_{dr} = \begin{bmatrix} (1,0) & (*,*) & (*,*) & \left(\left| \frac{\phi}{\beta} \right|_{dr}, 0 \right) \end{bmatrix}^T \quad (3.45)$$

where (\bullet, \bullet) denotes (magnitude, phase (degrees)).

Eigenvector Element Ratio Design Guidelines

In the previous section, desired lateral-directional eigenvectors were specified in terms of eigenvector element ratios ($|\phi/\beta|_{dr}$ and $(\beta/p)_{roll}$), and the adverse yaw ratio (N'_δ / L'_δ) . In this section, relationships between the Military Standard lateral-directional coupling criteria and these parameters will be developed. Using these relationships, the Military Standard Level One flying qualities boundaries can be translated into guidelines on eigenvector element ratios and the adverse yaw ratio.

Roll Rate Oscillation Criteria and Eigenvector Element Ratios

The Dutch roll contamination occurs primarily in roll rate when $|\phi/\beta|_{dr}$ is moderate-to-large. For these configurations, the Dutch roll contamination can be quantified in the time domain by the ratio (p_{osc} / p_{avg}) (equations (2.54-55)). This ratio can be expressed as a function of $|\phi/\beta|_{dr}$, $(\beta/p)_{roll}$, and (N'_δ / L'_δ) by defining it in terms of the system residues. This is done in the following.

The roll rate due to a unit step input in lateral stick expressed in terms of system residues and eigenvalues is given by (because there is only one input, the third subscript on the R 's has been omitted)

$$p(t) = \frac{R_{p,sprl}}{\lambda_{sprl}} e^{\lambda_{sprl} t} + \frac{R_{p,roll}}{\lambda_{roll}} e^{\lambda_{roll} t} + 2 \left| \frac{R_{p,dr}}{\lambda_{dr}} \right| e^{-\zeta_{dr} \omega_{dr} t} \cos(\omega_{dr} \sqrt{1 - \zeta_{dr}^2} t + \angle \frac{R_{p,dr}}{\lambda_{dr}}) \quad (3.46)$$

where (Churchhill et al. 1976)

$$R_{p,roll} = \frac{L_\delta \lambda_{roll} (\lambda_{roll} - z_\phi) (\lambda_{roll} - \bar{z}_\phi)}{(\lambda_{roll} - \lambda_{sprl}) (\lambda_{roll} - \lambda_{dr}) (\lambda_{roll} - \bar{\lambda}_{dr})} \quad (3.47)$$

$$R_{p,sprl} = \frac{L_{\delta} \lambda_{sprl} (\lambda_{sprl} - z_{\phi}) (\lambda_{sprl} - \bar{z}_{\phi})}{(\lambda_{sprl} - \lambda_{roll}) (\lambda_{sprl} - \lambda_{dr}) (\lambda_{sprl} - \bar{\lambda}_{dr})} \quad (3.48)$$

$$R_{p,dr} = \frac{L_{\delta} \lambda_{dr} (\lambda_{dr} - z_{\phi}) (\lambda_{dr} - \bar{z}_{\phi})}{(\lambda_{dr} - \lambda_{sprl}) (\lambda_{dr} - \lambda_{roll}) (\lambda_{dr} - \bar{\lambda}_{dr})} \quad (3.49)$$

and $z_{\phi} = -\omega_{\phi} \zeta_{\phi} + j\omega_{\phi} \sqrt{1 - \zeta_{\phi}^2}$ and \bar{x} denotes the complex conjugate of x . The p -to- δ_{stk} transfer function numerators z_{ϕ} can be expressed in terms of system eigenvalues, $|\phi/\beta|_{dr}$, $(\beta/p)_{roll}$, and $(N'_{\delta}/L'_{\delta})$ as

$$z_{\phi} \cong \tilde{z}_{\phi} = -\tilde{\omega}_{\phi} \tilde{\zeta}_{\phi} + j\tilde{\omega}_{\phi} \sqrt{1 - \tilde{\zeta}_{\phi}^2} \quad (3.50)$$

where

$$\omega_{\phi}^2 \cong \tilde{\omega}_{\phi}^2 = \omega_{dr}^2 \left(1 + \left(\frac{N'_{\delta}}{L'_{\delta}} \right) \left(\frac{\phi}{\beta} \right)_{dr} \right) \quad (3.51)$$

and

$$\begin{aligned} 2\omega_{\phi} \zeta_{\phi} \cong 2\tilde{\omega}_{\phi} \tilde{\zeta}_{\phi} = 2\omega_{dr} \zeta_{dr} + \\ \left(\frac{\phi}{\beta} \right)_{dr} \left\{ \left(\frac{N'_{\delta}}{L'_{\delta}} \right) (\lambda_{roll} + 2\zeta_{dr} \omega_{dr}) \right. \\ \left. + \left(\frac{\beta}{p} \right)_{roll} \frac{(\lambda_{roll}^2 + 2\lambda_{roll} \zeta_{dr} \omega_{dr} + \omega_{dr}^2)}{\left(1 - \lambda_{roll} \left(\frac{\beta}{p} \right)_{roll} \left(\frac{\phi}{\beta} \right)_{dr} \right)} \left(1 + \left(\frac{\phi}{\beta} \right)_{dr} \left(\frac{N'_{\delta}}{L'_{\delta}} \right) \right) \right\} \end{aligned} \quad (3.52)$$

where \tilde{x} denotes an approximation to x . An approximation to $p(t)$ expressed in terms of \tilde{z}_{ϕ} is given by

$$\tilde{p}(t) = \frac{\tilde{R}_{p,sprl}}{\lambda_{sprl}} e^{\lambda_{sprl} t} + \frac{\tilde{R}_{p,roll}}{\lambda_{roll}} e^{\lambda_{roll} t} + 2 \left| \frac{\tilde{R}_{p,dr}}{\lambda_{dr}} \right| e^{-\zeta_{dr} \omega_{dr} t} \cos(\omega_{dr} \sqrt{1 - \zeta_{dr}^2} t + \angle \frac{\tilde{R}_{p,dr}}{\lambda_{dr}}) \quad (3.53)$$

where

$$\tilde{R}_{p,roll} = \frac{L_{\delta} \lambda_{roll} (\lambda_{roll} - \tilde{z}_{\phi}) (\lambda_{roll} - \bar{\tilde{z}}_{\phi})}{(\lambda_{roll} - \lambda_{sprl}) (\lambda_{roll} - \lambda_{dr}) (\lambda_{roll} - \bar{\lambda}_{dr})} \quad (3.54)$$

$$\tilde{R}_{p,sprl} = \frac{L_{\delta} \lambda_{sprl} (\lambda_{sprl} - \tilde{z}_{\phi}) (\lambda_{sprl} - \bar{\tilde{z}}_{\phi})}{(\lambda_{sprl} - \lambda_{roll}) (\lambda_{sprl} - \lambda_{dr}) (\lambda_{sprl} - \bar{\lambda}_{dr})} \quad (3.55)$$

$$\tilde{R}_{p,dr} = \frac{L_{\delta} \lambda_{dr} (\lambda_{dr} - \tilde{z}_{\phi}) (\lambda_{dr} - \bar{\tilde{z}}_{\phi})}{(\lambda_{dr} - \lambda_{sprl}) (\lambda_{dr} - \lambda_{roll}) (\lambda_{dr} - \bar{\lambda}_{dr})} \quad (3.56)$$

The ratio (p_{osc}/p_{avg}) can be expressed as a function of $\tilde{p}(t)$ by

$$\frac{p_{osc}}{p_{avg}} \cong \frac{\tilde{p}_{osc}}{\tilde{p}_{avg}} = \frac{\tilde{p}_1 + \tilde{p}_3 - 2\tilde{p}_2}{\tilde{p}_1 + \tilde{p}_3 + 2\tilde{p}_2} \quad (3.57)$$

for ζ_{dr} less than or equal to 0.2 and

$$\frac{p_{osc}}{p_{avg}} \cong \frac{\tilde{p}_{osc}}{\tilde{p}_{avg}} = \frac{\tilde{p}_1 - \tilde{p}_2}{\tilde{p}_1 + \tilde{p}_2} \quad (3.58)$$

for ζ_{dr} greater than 0.2 where \tilde{p}_1 , \tilde{p}_2 , and \tilde{p}_3 are values of $\tilde{p}(t)$ at the first, second, and third peaks; respectively.

Equations (3.57-58) (along with equations (3.50-56)) provide expressions for ($\tilde{p}_{osc}/\tilde{p}_{avg}$) as a function of system eigenvalues, $|\phi/\beta|_{dr}$, $(\beta/p)_{roll}$, and $(N'\delta/L'\delta)$. Using these equations ($\tilde{p}_{osc}/\tilde{p}_{avg}$) can be calculated in the following way:

- 1) Choose system eigenvalues and values of $|\phi/\beta|_{dr}$, $(\beta/p)_{roll}$, and $(N'\delta/L'\delta)$. Note that this development requires $(1 + (N'\delta/L'\delta) |\phi/\beta|_{dr}) > 0$.
- 2) Calculate $\tilde{\omega}_{\phi}$ and $\tilde{\zeta}_{\phi}$ using equations (3.51-52) and form \tilde{z}_{ϕ} .
- 3) Calculate $\tilde{R}_{p,sprl}$, $\tilde{R}_{p,roll}$, and $\tilde{R}_{p,dr}$ using equations (3.54-56) and form $\tilde{p}(t)$.
- 4) Generate step time response using $\tilde{p}(t)$ (equation (3.53)).
- 5) Pick off peaks from $\tilde{p}(t)$ step time response (\tilde{p}_1 , \tilde{p}_2 , and \tilde{p}_3).
- 6) Calculate ($\tilde{p}_{osc}/\tilde{p}_{avg}$) using equation (3.57) or (3.58).

Sideslip Excursion Criteria and Eigenvector Element Ratios

The Dutch roll contamination occurs primarily in sideslip if $|\phi/\beta|_{dr}$ is low. The Dutch roll contamination can be quantified in the time domain by the ratio $(\Delta\beta_{max}/k_{\beta})$ (equations (2.58-59)). This parameter can be expressed as a function of $|\phi/\beta|_{dr}$, $(\beta/p)_{roll}$, and $(N'\delta/L'\delta)$ by defining it in terms of the system residues. This is done in the following.

The sideslip due to a step input in lateral stick expressed in terms of system residues and eigenvalues is given by (because there is only one input, the third subscript on the R 's has been omitted)

$$\begin{aligned}\beta(t) = \beta_0 + \frac{R_{\beta,sprl}}{\lambda_{sprl}} e^{\lambda_{sprl} t} + \frac{R_{\beta,roll}}{\lambda_{roll}} e^{\lambda_{roll} t} \\ + 2 \left| \frac{R_{\beta,dr}}{\lambda_{dr}} \right| e^{-\zeta_{dr} \omega_{dr} t} \cos(\omega_{dr} \sqrt{1 - \zeta_{dr}^2} t + \angle \frac{R_{\beta,dr}}{\lambda_{dr}})\end{aligned}\quad (3.59)$$

where

$$\beta_0 = - \left(\frac{R_{\beta,sprl}}{\lambda_{sprl}} + \frac{R_{\beta,roll}}{\lambda_{roll}} + 2 \left| \frac{R_{\beta,dr}}{\lambda_{dr}} \right| \cos(\angle \frac{R_{\beta,dr}}{\lambda_{dr}}) \right) \quad (3.60)$$

Making the assumptions of (2.12) and by specifying the desired spiral eigenvector as defined in (3.43), equation (3.59) reduces to

$$\beta(t) = \beta_0 + \frac{R_{\beta,roll}}{\lambda_{roll}} e^{\lambda_{roll} t} + 2 \left| \frac{R_{\beta,dr}}{\lambda_{dr}} \right| e^{-\zeta_{dr} \omega_{dr} t} \cos(\omega_{dr} \sqrt{1 - \zeta_{dr}^2} t + \angle \frac{R_{\beta,dr}}{\lambda_{dr}}) \quad (3.61)$$

where

$$\beta_0 = - \left(\frac{R_{\beta,roll}}{\lambda_{roll}} + 2 \left| \frac{R_{\beta,dr}}{\lambda_{dr}} \right| \cos(\angle \frac{R_{\beta,dr}}{\lambda_{dr}}) \right) \quad (3.62)$$

$$R_{\beta,roll} = \frac{-\dot{L}_\delta \left(\frac{\dot{N}_\delta}{\dot{L}_\delta} (\lambda_{roll} - \dot{L}_p) + \left(\dot{N}_p - \frac{g}{V_0} \right) \right)}{(\lambda_{roll} - \lambda_{dr})(\lambda_{roll} - \bar{\lambda}_{dr})} \quad (3.63)$$

$$R_{\beta,dr} = \frac{-\dot{L}_\delta \left(\frac{\dot{N}_\delta}{\dot{L}_\delta} (\lambda_{dr} - \dot{L}_p) + \left(\dot{N}_p - \frac{g}{V_0} \right) \right)}{(\lambda_{dr} - \lambda_{roll})(\lambda_{dr} - \bar{\lambda}_{dr})} \quad (3.64)$$

An approximation to $\beta(t)$ expressed in terms of $|\phi/\beta|_{dr}$, $(\beta/p)_{roll}$, and (N'_δ/L'_δ) is given by

$$\tilde{\beta}(t) = \tilde{\beta}_0 + \frac{\tilde{R}_{\beta,roll}}{\lambda_{roll}} e^{\lambda_{roll} t} + 2 \left| \frac{\tilde{R}_{\beta,dr}}{\lambda_{dr}} \right| e^{-\zeta_{dr} \omega_{dr} t} \cos(\omega_{dr} \sqrt{1 - \zeta_{dr}^2} t + \angle \frac{\tilde{R}_{\beta,dr}}{\lambda_{dr}}) \quad (3.65)$$

where

$$\tilde{\beta}_0 = - \left(\frac{\tilde{R}_{\beta,roll}}{\lambda_{roll}} + 2 \left| \frac{\tilde{R}_{\beta,dr}}{\lambda_{dr}} \right| \cos(\angle \frac{\tilde{R}_{\beta,dr}}{\lambda_{dr}}) \right) \quad (3.66)$$

$$\tilde{R}_{\beta,roll} = \frac{\dot{L}_{\delta} \left(1 + \frac{\dot{N}_{\delta}}{\dot{L}_{\delta}} \left(\frac{\phi}{\beta} \right)_{dr} \right) \left(\frac{\beta}{p} \right)_{roll}}{\left(1 - \lambda_{roll} \left(\frac{\beta}{p} \right)_{roll} \left(\frac{\phi}{\beta} \right)_{dr} \right)} \quad (3.67)$$

$$\tilde{R}_{\beta,dr} = \frac{-\dot{L}_{\delta} \left(\frac{\dot{N}_{\delta}}{\dot{L}_{\delta}} + (\lambda_{roll} - \bar{\lambda}_{dr}) \frac{\left(1 + \frac{\dot{N}_{\delta}}{\dot{L}_{\delta}} \left(\frac{\phi}{\beta} \right)_{dr} \right) \left(\frac{\beta}{p} \right)_{roll}}{\left(1 - \lambda_{roll} \left(\frac{\beta}{p} \right)_{roll} \left(\frac{\phi}{\beta} \right)_{dr} \right)} \right)}{(\lambda_{dr} - \bar{\lambda}_{dr})} \quad (3.68)$$

and \tilde{x} denotes an approximation to x . The ratio $(\Delta\beta_{\max} / k_{\beta})$ can be expressed as a function of $\tilde{\beta}(t)$ by

$$\frac{\Delta\beta_{\max}}{k_{\beta}} \cong \frac{\Delta\tilde{\beta}_{\max}}{\tilde{k}_{\beta}} \quad (3.69)$$

where

$$\Delta\beta_{\max} \cong \Delta\tilde{\beta}_{\max} = \max(\tilde{\beta}(t)) - \min(\tilde{\beta}(t)) \quad \text{for } 0 < t < t_{\beta} \quad (3.70)$$

where t_{β} is equal to 2 seconds or one half period of the Dutch roll, whichever is greater, and

$$k_{\beta} \cong \tilde{k}_{\beta} = \left. \frac{\tilde{\phi}(t)}{60} \right|_{t=1 \text{ sec}} \quad (3.71)$$

where

$$\begin{aligned} \tilde{\phi}(t) = & \tilde{\phi}_0 + \frac{\tilde{R}_{\phi,sprl}}{\lambda_{sprl}} e^{\lambda_{sprl} t} + \frac{\tilde{R}_{\phi,roll}}{\lambda_{roll}} e^{\lambda_{roll} t} \\ & + 2 \left| \frac{\tilde{R}_{\phi,dr}}{\lambda_{dr}} \right| e^{-\zeta_{dr} \omega_{dr} t} \cos(\omega_{dr} \sqrt{1 - \zeta_{dr}^2} t + \angle \frac{\tilde{R}_{\phi,dr}}{\lambda_{dr}}) \end{aligned} \quad (3.72)$$

with

$$\tilde{\phi}_0 = - \left(\frac{\tilde{R}_{\phi,sprl}}{\lambda_{sprl}} + \frac{\tilde{R}_{\phi,roll}}{\lambda_{roll}} + 2 \left| \frac{\tilde{R}_{\phi,dr}}{\lambda_{dr}} \right| \cos(\angle \frac{\tilde{R}_{\phi,dr}}{\lambda_{dr}}) \right) \quad (3.73)$$

$$\tilde{R}_{\phi,sprl} = \frac{\tilde{R}_{p,sprl}}{\lambda_{sprl}}; \quad \tilde{R}_{\phi,roll} = \frac{\tilde{R}_{p,roll}}{\lambda_{roll}}; \quad \tilde{R}_{\phi,dr} = \frac{\tilde{R}_{p,dr}}{\lambda_{dr}} \quad (3.74)$$

where $\tilde{R}_{p,sprl}$, $\tilde{R}_{p,roll}$, and $\tilde{R}_{p,dr}$ are defined by equations (3.54-56).

Equations (3.69-71) (along with equations (3.65-3.68) and (3.72-74)) provide expressions for $(\Delta\tilde{\beta}_{\max}/\tilde{k}_{\beta})$ as a function of system eigenvalues, eigenvector element ratios ($|\phi/\beta|_{dr}$ and $(\beta/p)_{roll}$), and $(N'\delta/L'\delta)$. Using these equations $(\Delta\tilde{\beta}_{\max}/\tilde{k}_{\beta})$ can be calculated in the following way:

- 1) Choose system eigenvalues and values of $|\phi/\beta|_{dr}$, $(\beta/p)_{roll}$, and $(N'\delta/L'\delta)$. Note that this development requires $(1 + (N'\delta/L'\delta) |\phi/\beta|_{dr}) > 0$.
- 2) Calculate $\tilde{\beta}_0$, $\tilde{R}_{\beta,roll}$, and $\tilde{R}_{\beta,dr}$ (using equations (3.66-68)) and form $\tilde{\beta}(t)$.
- 3) Generate step time response using $\tilde{\beta}(t)$ (equation (3.65)).
- 4) Calculate $\Delta\tilde{\beta}_{\max}$ (equation (3.70)) from $\tilde{\beta}(t)$ step time response.
- 5) Calculate $\tilde{\phi}_0$, $\tilde{R}_{\phi,sprl}$, $\tilde{R}_{\phi,roll}$, and $\tilde{R}_{\phi,dr}$ (using equations (3.73-74)) and form $\tilde{\phi}(t)$.
- 6) Calculate \tilde{k}_{β} (equation (3.71)) from $\tilde{\phi}(t)$ (equation (3.72)).
- 7) Calculate $(\Delta\tilde{\beta}_{\max}/\tilde{k}_{\beta})$ using equation (3.69).

Phase Angle of Dutch Roll Component of Sideslip and Eigenvector Element Ratios

The values of (p_{osc}/p_{avg}) and $(\Delta\beta_{\max}/k_{\beta})$ that a pilot will accept are a function of the phase angle of the Dutch roll component of sideslip, Ψ_{β} . This angle is directly related to the angular position of the zero relative to the Dutch roll pole in the roll rate-to-lateral stick transfer function Ψ_1 . This angle is given by

$$\Psi_1 = \text{atan} \left(\frac{\omega_{dr} \sqrt{1 - \zeta_{dr}^2} - \omega_{\phi} \sqrt{1 - \zeta_{\phi}^2}}{-\zeta_{dr} \omega_{dr} + \zeta_{\phi} \omega_{\phi}} \right) \quad (3.75)$$

As shown earlier (equation (2.56)), for positive dihedral, Ψ_{β} and Ψ_1 can be related by (Chalk et al. 1969)

$$\Psi_{\beta} \equiv \Psi_1 - 270 \text{ (degrees)} \quad (3.76)$$

An approximation to Ψ_{β} expressed in terms of system eigenvalues, $|\phi/\beta|_{dr}$, $(\beta/p)_{roll}$, and $(N'\delta/L'\delta)$ is given by

$$\Psi_{\beta} \equiv \tilde{\Psi}_{\beta} = \text{atan} \left(\frac{\omega_{dr} \sqrt{1 - \zeta_{dr}^2} - \tilde{\omega}_{\phi} \sqrt{1 - \tilde{\zeta}_{\phi}^2}}{-\zeta_{dr} \omega_{dr} + \tilde{\zeta}_{\phi} \tilde{\omega}_{\phi}} \right) - 270 \quad (3.77)$$

where $\tilde{\omega}_{\phi}$ and $\tilde{\zeta}_{\phi}$ are defined by equations (3.51) and (3.52).

Equation (3.77) provides an expression for $\tilde{\Psi}_\beta$ as a function of system eigenvalues, $|\phi/\beta|_{dr}$, $(\beta/p)_{roll}$, and $(N'\delta/L'\delta)$. Using these equations $\tilde{\Psi}_\beta$ can be calculated in the following way:

- 1) Choose system eigenvalues and values of $|\phi/\beta|_{dr}$, $(\beta/p)_{roll}$, and $(N'\delta/L'\delta)$. Note that this development requires $(1 + (N'\delta/L'\delta) |\phi/\beta|_{dr}) > 0$.
- 2) Calculate $\tilde{\omega}_\phi$ and $\tilde{\zeta}_\phi$ using equations (3.51-52).
- 3) Calculate $\tilde{\Psi}_\beta$ using equation (3.77).

This development assumes values of $|\phi/\beta|_{dr}$, $(\beta/p)_{roll}$, and $(N'\delta/L'\delta)$ have been chosen that yield complex conjugate transfer function zeros (i.e. $-1 < \tilde{\zeta}_\phi < 1$ and $\tilde{\omega}_\phi > 0$).

Generating Eigenspace Flying Qualities Guidelines

Eigenspace flying qualities guidelines for choosing $(\beta/p)_{roll}$ and $(N'\delta/L'\delta)$ for a given value of $|\phi/\beta|_{dr}$ are determined in the following way:

- 1) Choose value of $|\phi/\beta|_{dr}$ and desired eigenvalues (roll and spiral eigenvalues at Level One locations).
- 2) Evaluate $(\tilde{p}_{osc}/\tilde{p}_{avg})$, $(\Delta\tilde{\beta}_{max}/\tilde{k}_\beta)$, and $\tilde{\Psi}_\beta$ over desired range of $(\beta/p)_{roll}$ and $N\delta/L\delta$.
- 3) The eigenspace roll rate oscillation guideline is based on the Military Standard roll rate oscillation criteria. This guideline can be determined by overlaying the $(\tilde{p}_{osc}/\tilde{p}_{avg})$ and $\tilde{\Psi}_\beta$ data and translating the Level One roll rate oscillation boundary into boundaries on $(\beta/p)_{roll}$ and $(N'\delta/L'\delta)$.
- 4) The eigenspace sideslip excursion guideline is based on the Military Standard sideslip excursion criteria. This guideline can be determined by overlaying the $(\Delta\tilde{\beta}_{max}/\tilde{k}_\beta)$ and $\tilde{\Psi}_\beta$ data and translating the Level One sideslip excursion boundary into boundaries on $(\beta/p)_{roll}$ and $(N'\delta/L'\delta)$.

The appropriate guideline to use is a function of the value of $|\phi/\beta|_{dr}$. For low $|\phi/\beta|_{dr}$ (less than approximately 1.5), the eigenspace sideslip excursion guideline should be used. For moderate-to-high $|\phi/\beta|_{dr}$ (greater than approximately 5), the eigenspace roll rate oscillation guideline should be used. For intermediate values of $|\phi/\beta|_{dr}$, both guidelines must be satisfied to meet Level One flying qualities. A composite eigenspace roll rate oscillation/sideslip excursion guideline can be obtained by overlaying the guidelines from these two criteria.

Eigenspace Flying Qualities Guideline Examples

Example guidelines are generated for a moderate-to-large, intermediate, and a low value of $|\phi/\beta|_{dr}$. These guidelines are developed for values of $|\phi/\beta|_{dr}$, $(\beta/p)_{roll}$, and $(N'\delta/L'\delta)$ that yield complex conjugate transfer function zeros in the left half plane (i.e. $0 < \tilde{\zeta}_\phi < 1$ and $\tilde{\omega}_\phi > 0$). For all of these cases the eigenvalues are set to the following values:

$\lambda_{sprl} = -0.005$, $\lambda_{roll} = -2.5$, $\lambda_{dr} = (\omega_{dr} = 2.0 \text{ (rad/sec)}, \zeta_{dr} = 0.1)$. The values of $|\phi/\beta|_{dr}$ considered are: $|\phi/\beta|_{dr} = 10$, $|\phi/\beta|_{dr} = 5$, and $|\phi/\beta|_{dr} = 1$.

Example One – $|\phi/\beta|_{dr} = 10$

The roll rate oscillation criteria is the suggested primary criteria for moderate-to-large $|\phi/\beta|_{dr}$. The first step in generating the eigenspace roll rate oscillation guideline is to evaluate $(\tilde{p}_{osc}/\tilde{p}_{avg})$ and $\tilde{\Psi}_\beta$ over the desired range of $(\beta/p)_{roll}$ and $(N'\delta/L'\delta)$. Figure 7 shows how lines of constant $(\tilde{p}_{osc}/\tilde{p}_{avg})$ are a function of $(\beta/p)_{roll}$ and $(N'\delta/L'\delta)$ for $|\phi/\beta|_{dr} = 10$. This figure shows contour lines for $(\tilde{p}_{osc}/\tilde{p}_{avg})$ equal to 0.05 and 0.25. The magnitude of $(\tilde{p}_{osc}/\tilde{p}_{avg})$ is directly related to the cancellation of the Dutch roll pole in the roll rate-to-lateral stick transfer function. At $(\beta/p)_{roll} = 0$ and $(N'\delta/L'\delta) = 0$, the Dutch roll pole is canceled and $(\tilde{p}_{osc}/\tilde{p}_{avg})$ equals zero. As $(\beta/p)_{roll}$ and $(N'\delta/L'\delta)$ increase in magnitude, there is an increase in the Dutch roll modes contribution to the roll rate response and therefore $(\tilde{p}_{osc}/\tilde{p}_{avg})$ increases in magnitude.

Figure 8 shows how lines of constant $\tilde{\Psi}_\beta$ are a function of $(\beta/p)_{roll}$ and $(N'\delta/L'\delta)$ for a value of $|\phi/\beta|_{dr} = 10$. This figure shows contour lines for $\tilde{\Psi}_\beta$ equal to 0, -130, -200, -270, and -340 degrees. Since $\tilde{\Psi}_\beta$ is directly related to the angular position of the zero relative to the Dutch roll pole in the roll rate-to-lateral stick transfer function, the constant $\tilde{\Psi}_\beta$ lines radiate from the point $(\beta/p)_{roll} = 0$, $(N'\delta/L'\delta) = 0$. By overlaying the $(\tilde{p}_{osc}/\tilde{p}_{avg})$ and $\tilde{\Psi}_\beta$ contour data, the Military Standard Level One roll rate oscillation criteria can be translated into boundaries on $(\beta/p)_{roll}$ and $(N'\delta/L'\delta)$. This is done in Figure 9. This figure shows how the four line segments that compose the Military Standard criteria map into the eigenspace guideline. The result of this mapping is given by the solid black line. Values of $(\beta/p)_{roll}$ and $(N'\delta/L'\delta)$ that lie inside this boundary meet this criteria. This guideline is presented again in Figure 10 without the $(\tilde{p}_{osc}/\tilde{p}_{avg})$ and $\tilde{\Psi}_\beta$ contours.

Example Two – $|\phi/\beta|_{dr} = 5$

For intermediate values of $|\phi/\beta|_{dr}$, both guidelines must be satisfied to meet Level One flying qualities. The eigenspace guidelines are generated by first evaluating $(\tilde{p}_{osc}/\tilde{p}_{avg})$, $(\Delta\tilde{\beta}_{max}/\tilde{k}_\beta)$, and $\tilde{\Psi}_\beta$ over the desired range of $(\beta/p)_{roll}$ and $(N'\delta/L'\delta)$. The eigenspace roll rate oscillation guideline is obtained by overlaying the $(\tilde{p}_{osc}/\tilde{p}_{avg})$ and $\tilde{\Psi}_\beta$ data and translating the Military Standard Level One roll rate oscillation boundary into boundaries on $(\beta/p)_{roll}$ and $(N'\delta/L'\delta)$. This guideline is given by the solid black line in Figure 11. The eigenspace sideslip excursion guideline is generated by overlaying the $(\Delta\tilde{\beta}_{max}/\tilde{k}_\beta)$ and $\tilde{\Psi}_\beta$ data and translating the Military Standard Level One sideslip excursion boundary into boundaries on $(\beta/p)_{roll}$ and $(N'\delta/L'\delta)$. This guideline is given by the solid gray line in Figure 12.

Both of these criteria must be satisfied to meet Level One flying qualities. The composite eigenspace guideline is obtained by overlaying these two criteria. This is done in Figure 13. The eigenspace roll rate oscillation guideline is given by the solid black line and

the eigenspace sideslip excursion guideline is given by the solid gray line. Values of $(\beta/p)_{roll}$ and $(N'\delta/L'\delta)$ that lie inside both of these boundaries will yield Level One flying qualities. As can be seen, for this value of $|\phi/\beta|_{dr}$ both guidelines are approximately the same size, but over most of the boundary the roll rate oscillation guideline is the more restrictive criteria.

As can be seen by comparing Figures 10 and 11, the eigenspace roll rate oscillation guideline is approximately the same shape as for $|\phi/\beta|_{dr}=10$ but considerably larger in size. This demonstrates the sensitivity of this guideline to the value of $|\phi/\beta|_{dr}$. As $|\phi/\beta|_{dr}$ decreases, the system's flying qualities become less sensitive to variations in $(\beta/p)_{roll}$ and $(N'\delta/L'\delta)$.

Example Three – $|\phi/\beta|_{dr} = 1$

The sideslip excursion criteria is the suggested primary criteria for low $|\phi/\beta|_{dr}$. The eigenspace sideslip excursion guideline is generated by first evaluating $(\Delta\tilde{\beta}_{max}/\tilde{k}_\beta)$ and $\tilde{\Psi}_\beta$ over the desired range of $(\beta/p)_{roll}$ and $(N'\delta/L'\delta)$. The guideline is obtained by overlaying the $(\Delta\tilde{\beta}_{max}/\tilde{k}_\beta)$ and $\tilde{\Psi}_\beta$ data and translating the Military Standard Level One sideslip excursion boundary into boundaries on $(\beta/p)_{roll}$ and $(N'\delta/L'\delta)$. This guideline is given by the solid gray line in Figure 14. Values of $(\beta/p)_{roll}$ and $(N'\delta/L'\delta)$ that lie inside this boundary meet this criteria.

As can be seen by comparing Figures 12 and 14, this guideline is approximately the same size as for $|\phi/\beta|_{dr} = 5$. This guideline is considerably less sensitive to variations in the value of $|\phi/\beta|_{dr}$ than the roll rate oscillation guideline.

4.0 CONCLUDING REMARKS

This report presents the development of lateral-directional flying qualities guidelines with application to eigenspace assignment methods. These guidelines will assist designers in choosing eigenvectors to achieve desired closed-loop flying qualities or performing trade-offs between flying qualities and other important design requirements, such as, achieving realizable gain magnitudes or desired system robustness. This has been accomplished by developing relationships between the system's eigenvectors and the roll rate and sideslip transfer functions. Using these relationships, along with constraints imposed by system dynamics, key eigenvector elements have been identified and guidelines for choosing values of these elements to yield desirable flying qualities developed.

Two guidelines are developed - one for low $|\phi/\beta|_{dr}$ and one for moderate-to-high $|\phi/\beta|_{dr}$. These flying qualities guidelines are based upon the Military Standard lateral-directional coupling criteria for high performance aircraft. The low $|\phi/\beta|_{dr}$ eigenspace guideline is based on the sideslip excursion criteria. The high $|\phi/\beta|_{dr}$ eigenspace guideline is based on the roll rate oscillation criteria. For intermediate values of $|\phi/\beta|_{dr}$, both guidelines must be satisfied to meet desired flying qualities. A composite eigenspace roll rate oscillation/sideslip excursion guideline is obtained by overlaying the guidelines from these two criteria.

Example guidelines are generated for a large, an intermediate, and low value of $|\phi/\beta|_{dr}$. For all of these cases the eigenvalues are set to fixed values and are not varied. In these examples it was shown that the value of the ratio $|\phi/\beta|_{dr}$ has a strong effect on the

eigenspace roll rate oscillation guideline, whereas the eigenspace sideslip excursion guideline is relatively insensitive to variations in $|\phi/\beta|_{dr}$.

Piloted simulation flying qualities experiments are planned to validate and refine these guidelines.

5.0 REFERENCES

Aeronautical Systems Division, Air Force Systems Command, "Military Specification - Flying Qualities of Piloted Vehicles," MIL-STD-1797A, January 1990.

Andry, A.N., Shapiro, E.Y., and Chung, J.C., "On Eigenstructure Assignment for Linear Systems," IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-19, No. 5, September 1983.

Brogan, W. L., "Modern Control Theory," Quantum Publishers, Inc., New York, 1974.

Buttrill, C.S., Arbuckle, P.D., and Hoffler, K.D., "Simulation Model of a Twin-Tail, High Performance Airplane," NASA TM-107601, July 1992.

Chalk, C.R., Neal, T.P., Harris, T.M., Pritchard, F.E., and Woodcock, R.J., "Background Information and User Guide for MIL-F-8785B(ASG), Military Specification - Flying Qualities of Piloted Airplanes," AFFDL-TR-69-72, August 1969.

Chalk, C.R., DiFranco, D.A., Lebacqz, J.V., and Neal, T.P., "Revisions to MIL-F-8785B(ASG) Proposed by Cornell Aeronautical Laboratory Under Contract F33615-71-C-1254," AFFDL-TR-72-41, April 1973.

Churchhill, R.V., Brown, J.W., and Verhey R.F., "Complex Variables and Applications," McGraw-Hill Book Company, 1976.

Cooper, G.E., "Understanding and Interpreting Pilot Opinion," Aeronautical Engineering Review, Volume 16, March 1957.

Cooper G.E., and Harper, R.P. Jr., "The Use of Pilot Rating in the Evaluation of Aircraft Handling Qualities," NASA TN D-5253, April 1969.

Costigan, M.J., and Calico, R.A., "An Analysis of Lateral-Directional Handling Qualities and Eigenstructure of High Performance Aircraft," AIAA-89-0017, 1989.

Cunningham, T.B., "Eigenspace Selection Procedures for Closed-Loop Response Shaping with Modal Control," Proceedings of the IEEE Conference on Decision and Control, Albuquerque, NM, December 1980.

Davidson, J.B., and Schmidt, D.K., "Flight Control Synthesis for Flexible Aircraft using Eigenspace Assignment," NASA CR-178164, June 1986.

Davidson, J., Foster, J., Ostroff, A., Lallman, F., Murphy, P., Hoffler, K., and Messina, M., "Development of a Control Law Design Process Utilizing Advanced Synthesis Methods with Application to the NASA F-18 HARV," High Alpha Projects and Technology Conference, Dryden Flight Research Facility, NASA CP-3137, April 21-23, 1992.

Davidson, J.B., and Andrisani, D., "Gain Weighted Eigenspace Assignment," NASA TM-109130, May 1994.

McRuer, D., Ashkenas, I., and Graham, D., "Aircraft Dynamics and Automatic Control," Princeton University Press, 1973.

Meeker, J.I., and Hall, G.W., "In-Flight Evaluation of Lateral-Directional Handling Qualities for the Fighter Mission," AFFDL-TR-67-98, October 1967.

Moore, B.C., "On the Flexibility Offered by State Feedback in Multivariable Systems Beyond Closed-Loop Eigenvalue Assignment," IEEE Transactions on Automatic Control, Vol. AC-21, October 1976.

Murphy, P., Hoffler, K., Davidson, J., Ostroff, A. Lallman, F., and Messina, M., "Preliminary Evaluation of HARV NASA-1 Control Law Flight Test Results," Fourth High Alpha Conference, NASA Dryden Flight Research Center, NASA CP-10143, July 12-14, 1994.

Srinathkumar, S., "Eigenvalue/Eigenvector Assignment Using Output Feedback," IEEE Transactions on Automatic Control, Volume AC-23, Number 1, February 1978.

Appendix - Primary Lateral-Directional Coupling Derivatives

A coupling derivative is one in which a motion, angle, or control input about one axis imparts a moment about an orthogonal axis. The primary lateral-directional coupling derivatives are: roll moment due to sideslip angle L_β , roll moment due to yaw rate L_r , yaw moment due to roll rate N_p , and yaw moment due to lateral controls N_δ .

Adverse Yaw due to Ailerons N_δ

In an aircraft in which ailerons are the primary lateral control effector, a right roll control input results in the left aileron down and right aileron up. This leads to a rolling moment due to more lift on the left wing and less on the right. More lift on the left wing results in more drag on the left wing. Therefore, there is also a yaw moment applied to the aircraft. For ailerons this yaw moment is opposite to the turn (adverse).

Dihedral Effect L_β

Dihedral of the open-loop airframe is primarily affected by wing location (mid-wing, high-wing, etc.), dihedral angle, and sweep. When an aircraft with positive dihedral encounters a positive sideslip it will tend to roll to the left because the right wing will see a higher angle-of-attack. An aircraft with positive dihedral effect will cause an aircraft to roll away from the sideslip. A negative value of L_β is positive dihedral.

Roll Moment due to Yaw Rate L_r

When an aircraft rotates to the right, the left wing will see an increase in forward velocity (and the right a decrease) due to the rotation. This velocity change results in lift changes that cause a roll moment in the direction of the yaw rate. In addition, the yaw rate generates a lateral velocity change at the tail. This results in a side force at the tail (which is usually above the roll axis) causing a roll moment.

Yaw Moment due to Roll Rate N_p

The primary contribution to N_p is due to the wing. This derivative is a component of adverse yaw. When an aircraft rolls to the left, an angle-of-attack increment is generated due to the roll rate. This angle-of-attack increment increases the lift on the downward wing (and decreases on the upward wing) and results in the lift vector being tilted forward. This results in a yaw moment usually opposite to the roll.

Primed Derivatives

The primed derivatives are defined by

$$\dot{L}_i = \frac{L_i + (I_{xz}/I_x)N_i}{1 - (I_{xz}^2/I_x I_z)}; \quad \dot{N}_i = \frac{N_i + (I_{xz}/I_z)L_i}{1 - (I_{xz}^2/I_x I_z)} \quad (\text{A.1})$$

where the subscript i denotes a motion or input quantity.

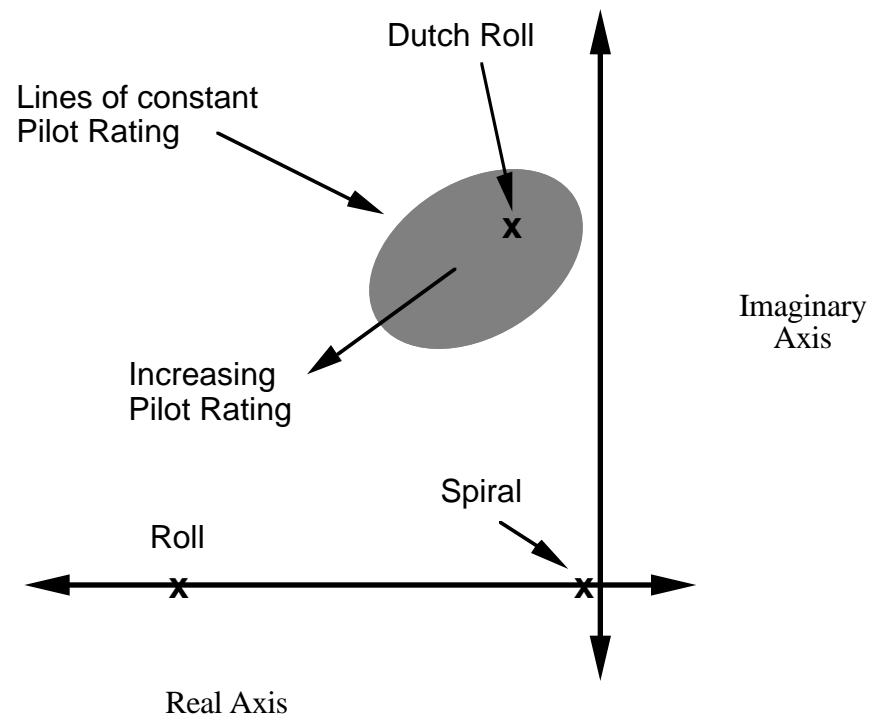


Figure 1 - Areas of acceptable zero locations p-to-lateral stick transfer function.

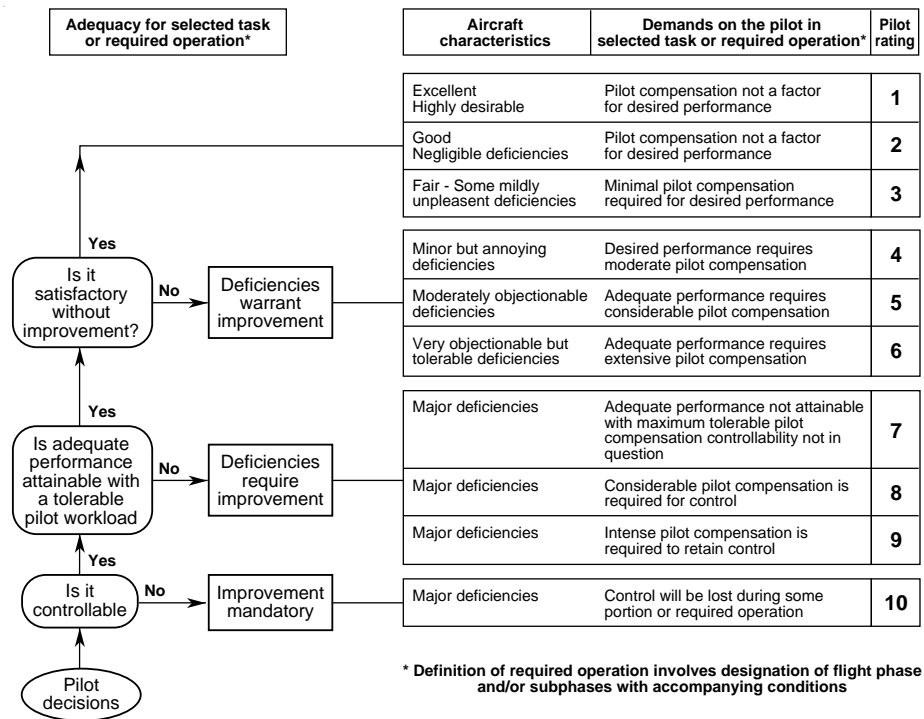


Figure 2 - Cooper-Harper handling qualities rating scale (Cooper and Harper 1969).

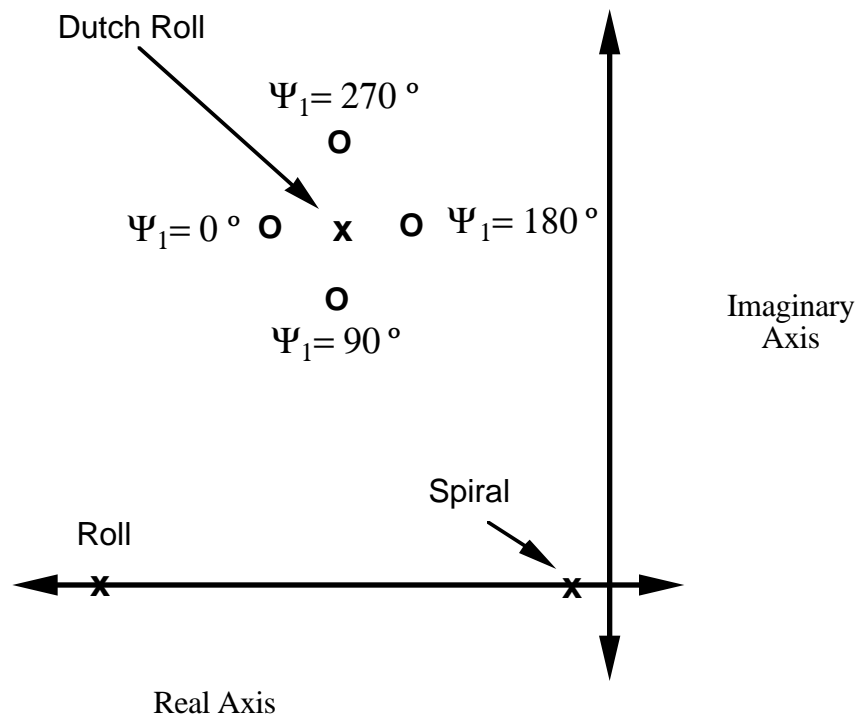


Figure 3 - Values of Ψ_1 for various zero locations p-to-lateral stick transfer function.

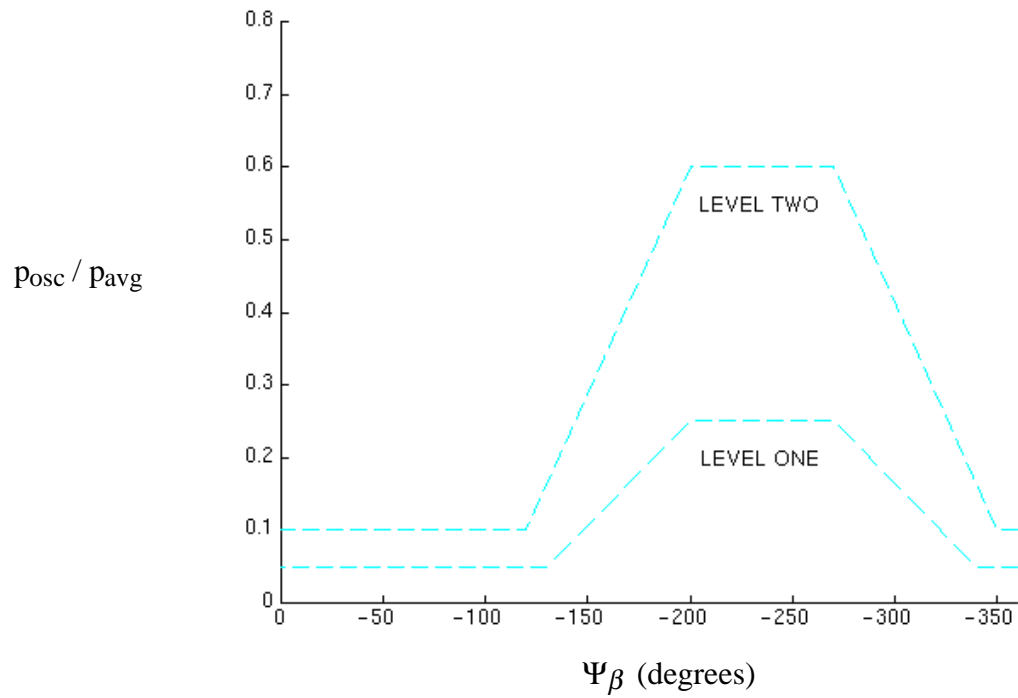


Figure 4 - (p_{osc} / p_{avg}) Roll Oscillation Criteria for Positive Dihedral.

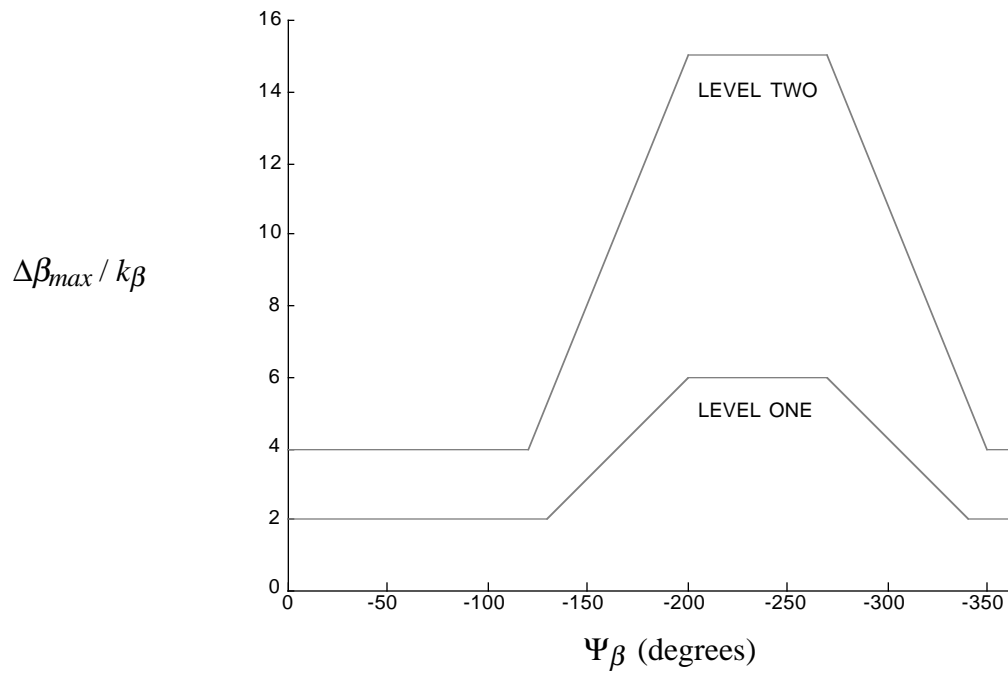


Figure 5 - ($\Delta\beta_{max} / k\beta$) Sideslip Excursion Criteria for Positive Dihedral.

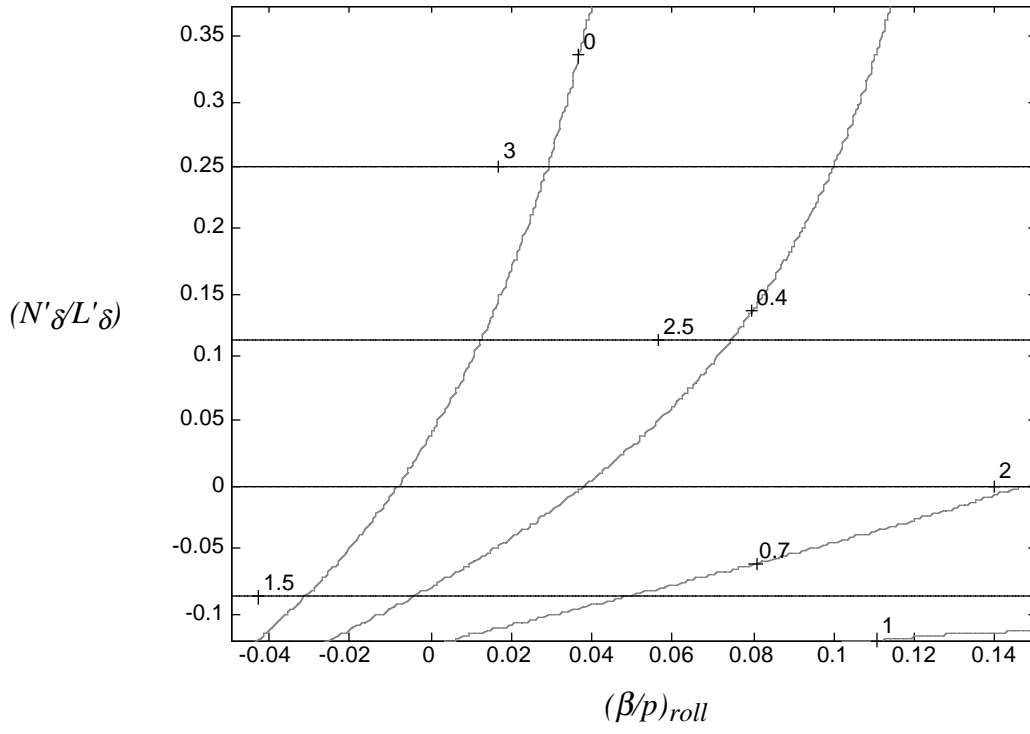


Figure 6 - Lines of Constant ω_ϕ (solid lines (rad/sec)) and ζ_ϕ (dashed lines) for $|\phi/\beta|_{dr} = 5$.

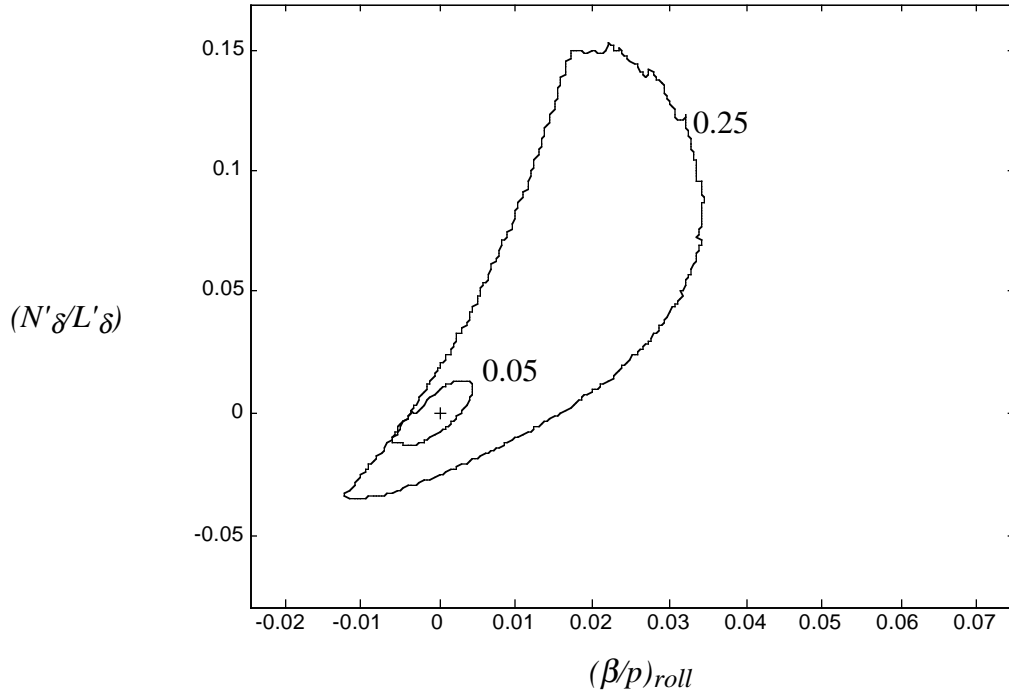


Figure 7 - Lines of Constant $(\tilde{p}_{osc} / \tilde{p}_{av})$ for $|\phi/\beta|_{dr} = 10$.

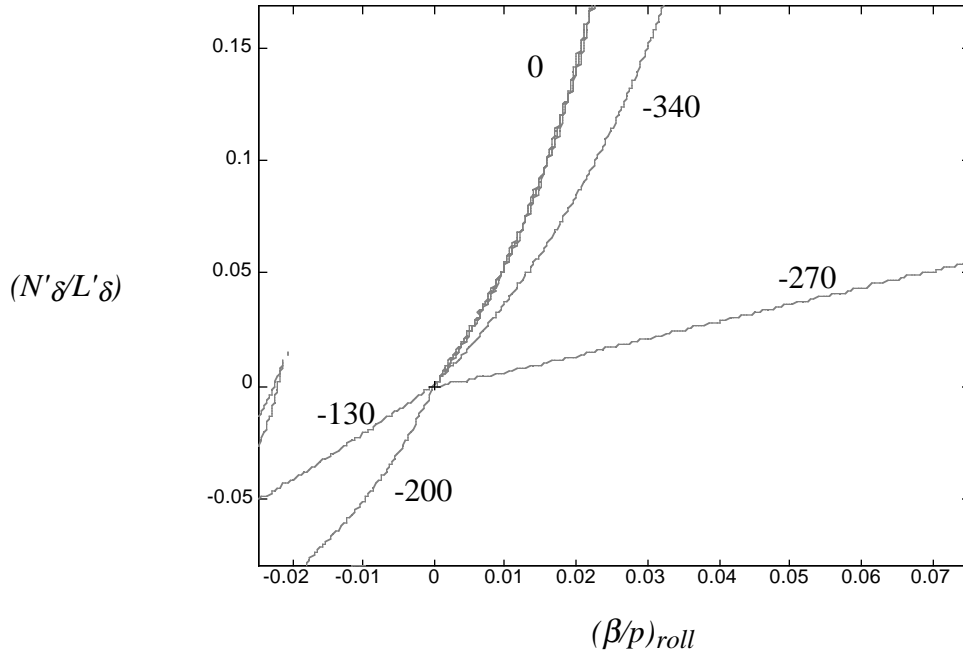


Figure 8 - Lines of Constant $\tilde{\Psi}_\beta$ (degrees) for $|\phi/\beta|_{dr} = 10$.

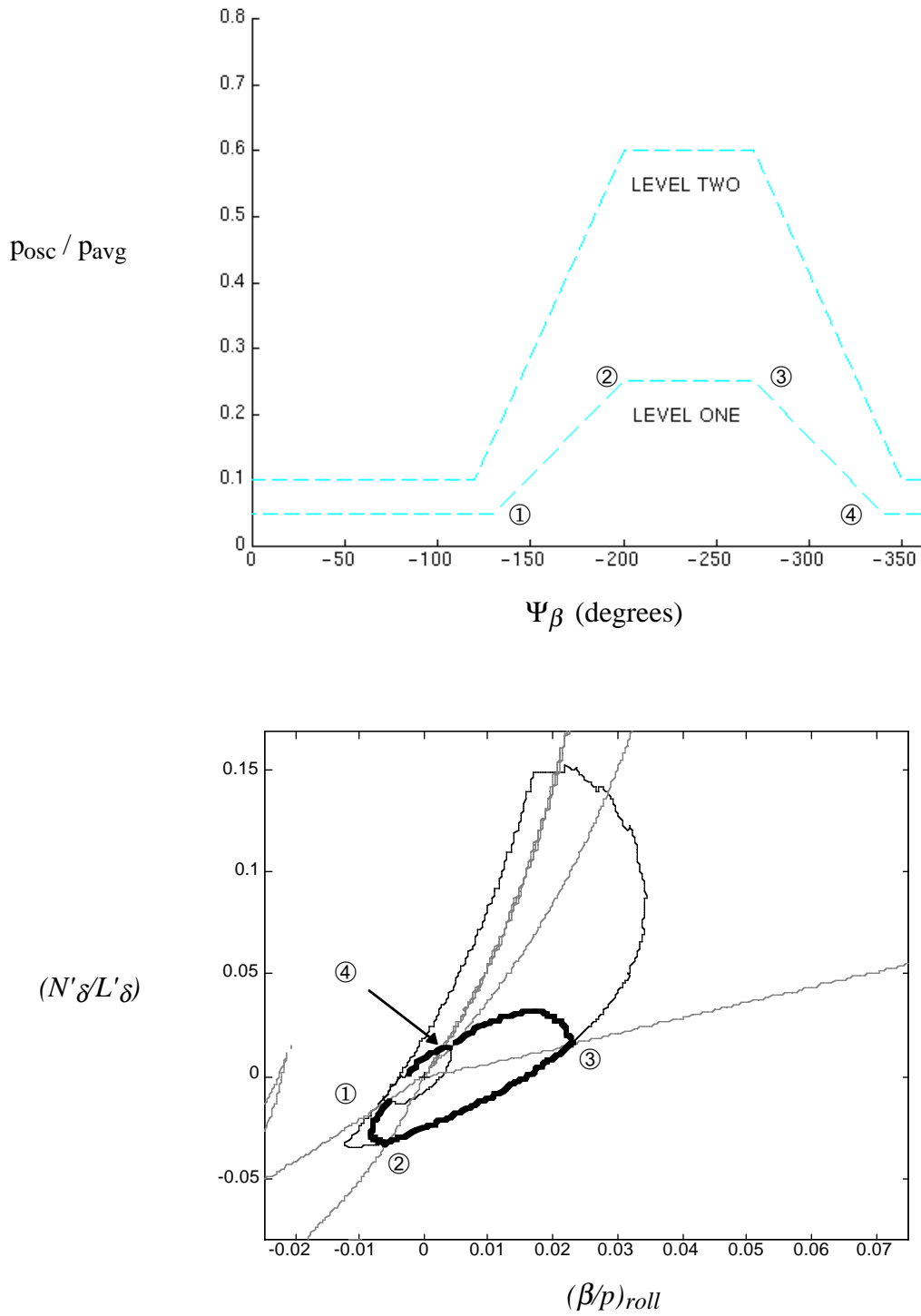


Figure 9 - Mapping Level One Military Standard Roll Rate Oscillation Criteria into Eigenspace Guideline for $|\phi/\beta|_{dr} = 10$.

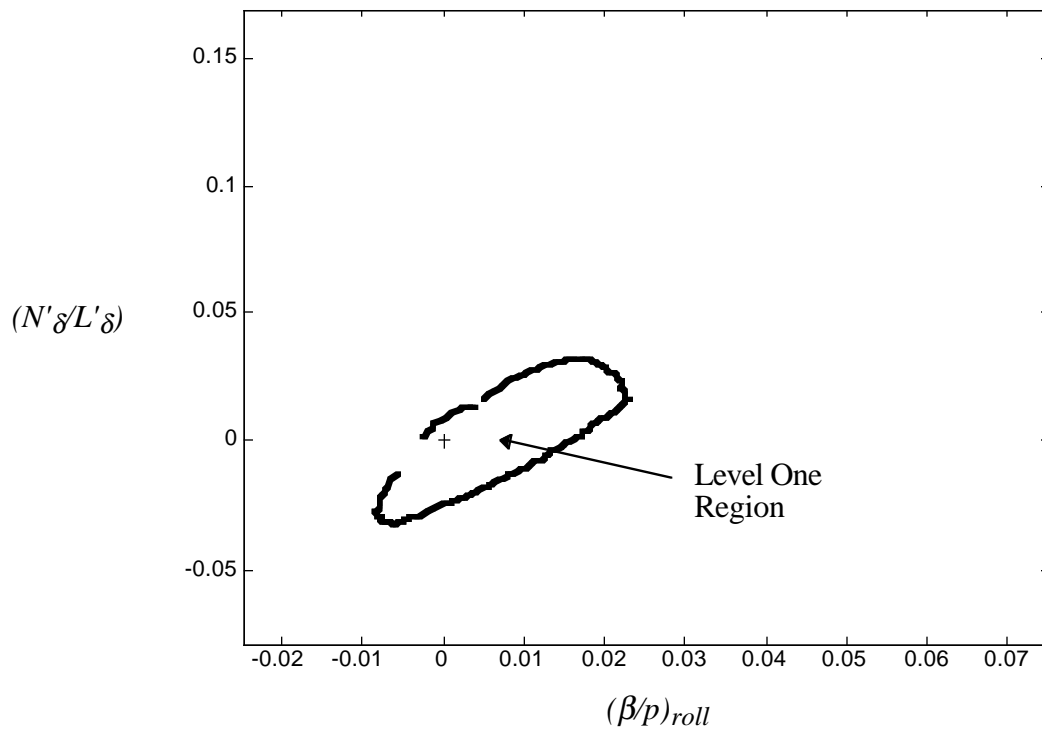


Figure 10 - Level One Eigenspace Roll Rate Oscillation Guideline for $|\phi/\beta|_{dr} = 10$.

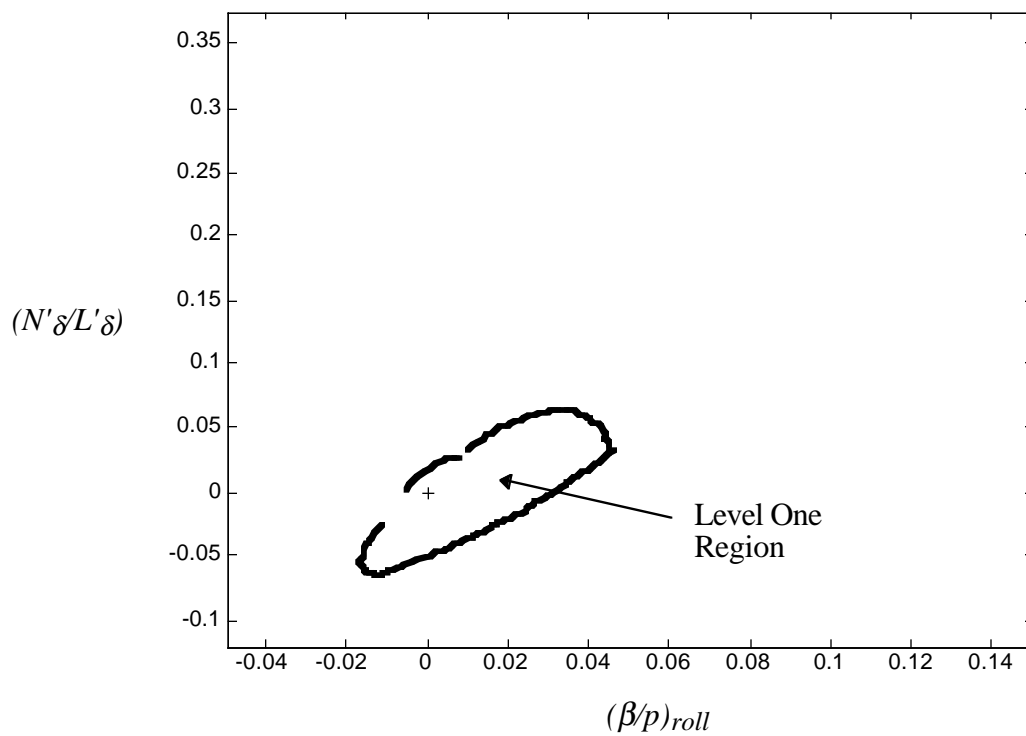


Figure 11 - Level One Eigenspace Roll Rate Oscillation Guideline for $|\phi/\beta|_{dr} = 5$.

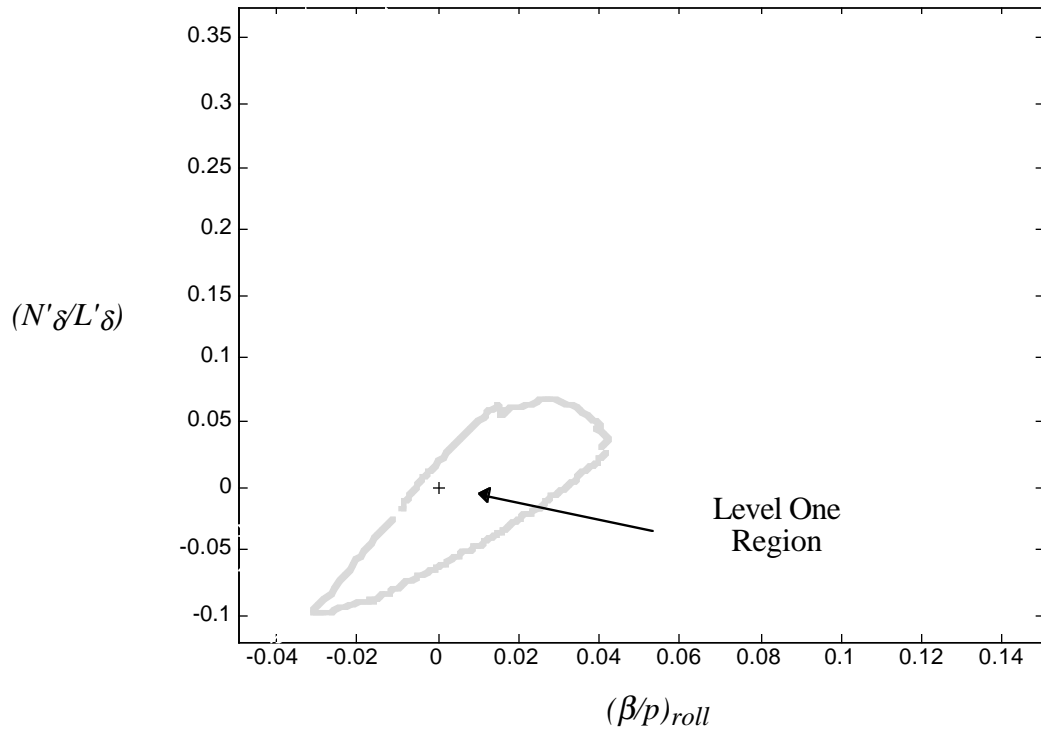


Figure 12 - Level One Eigenspace Sideslip Excursion Guideline for $|\phi/\beta|_{dr} = 5$.

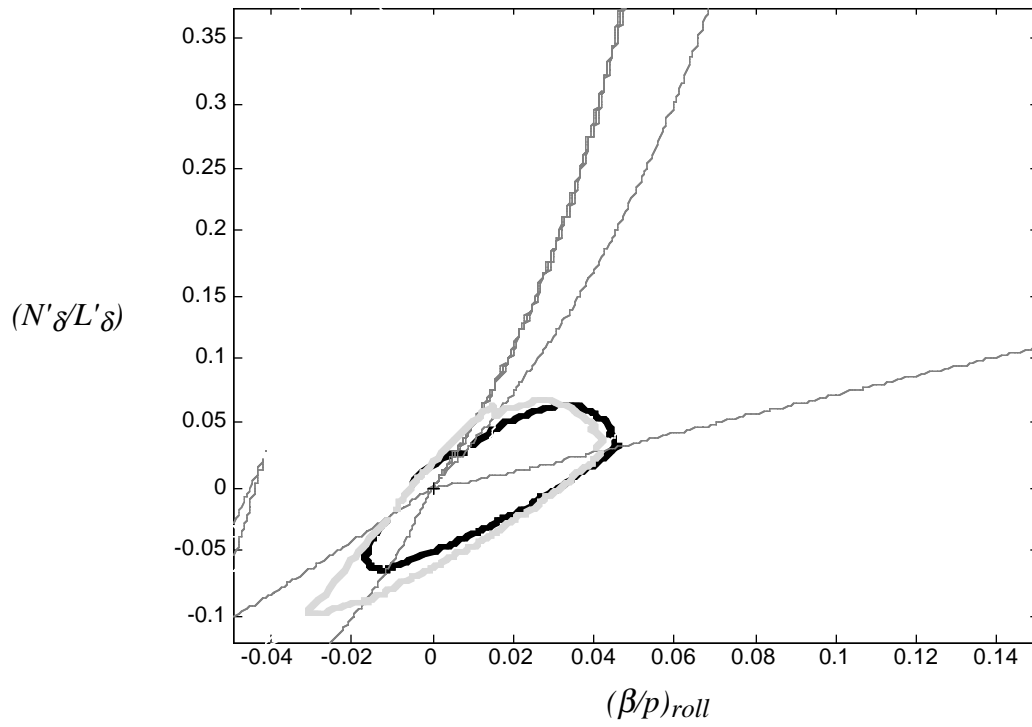


Figure 13 - Overlay of Eigenspace Roll Rate Oscillation (black line) and Eigenspace Sideslip Excursion (gray line) Flying Qualities Guidelines for $|\phi/\beta|_{dr} = 5$.

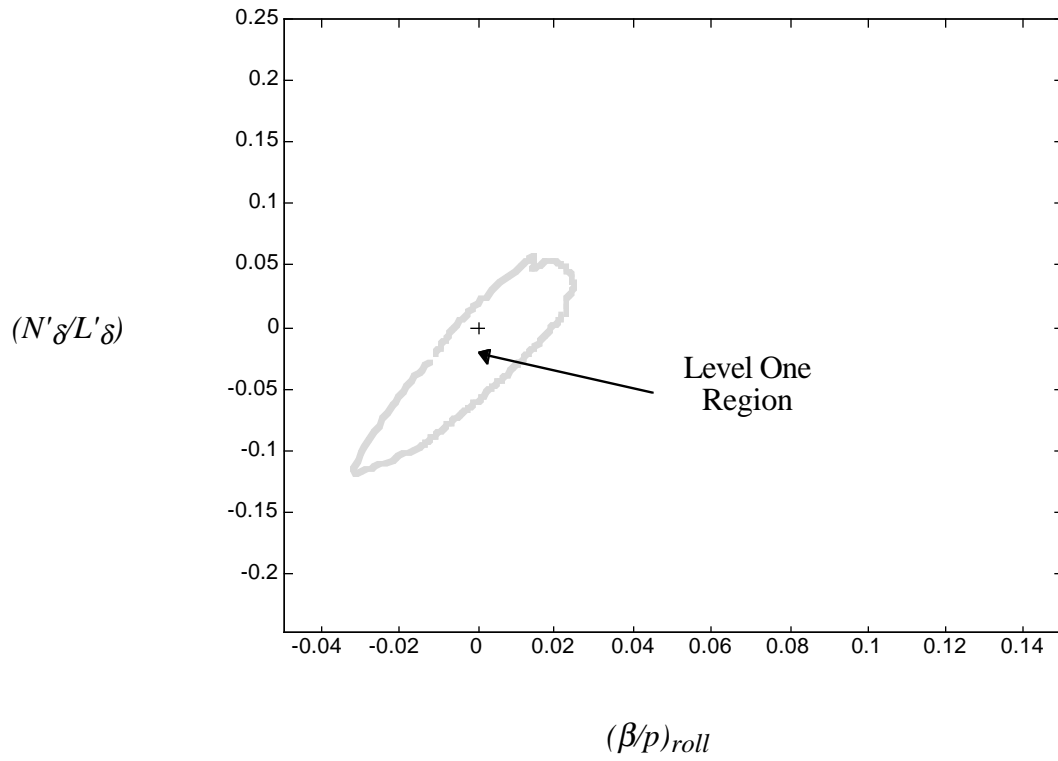


Figure 14 - Level One Eigenspace Sideslip Excursion Guideline for $|\phi/\beta|_{dr} = 1$.

	AB-1	AB-2	AB-3
ω_{dr} (rad/sec)	2.36	2.51	2.55
ζ_{dr}	0.1	0.1	0.1
$ \phi/\beta _{dr}$	1.5	1.5	1.5
$\angle(\phi/\beta)_{dr}$ (degrees)	50	44	47
τ_{roll} (seconds)	0.4	0.4	0.4
τ_{spiral} (seconds)	1000	1000	1000

Table 1 - Modal Data for Group AB.
(Data obtained and derived from Chalk 1969 and Chalk 1973)

Sub-Group	Config.	ω_ϕ (rad/sec)	ζ_ϕ	(p_{osc}/p_{avg})	$(\Delta\beta_{max}/k\beta)$ (degrees)	Cooper Rating
AB-1	a	2.02	0.04	0.144	17.9	7
	b	2.08	0.05	0.105	14.2	7
	c	2.16	0.05	0.07	9.76	7
	d	2.28	0.06	0.034	4.58	6.5
	e	2.36	0.05	0.024	4.16	7
	f	2.42	0.07	0.028	5.47	8
AB-2	a	1.97	0.10	0.265	7.65	7
	b	2.12	0.10	0.15	5.34	4
	c	2.34	0.10	0.044	2.37	3
	d	2.40	0.10	0.026	1.62	4
	e	2.44	0.10	0.015	1.05	3
	f	2.48	0.12	0.003	0.45	3
	g	-	-	0.013	0.53	3
	h	2.66	0.11	0.038	1.54	4.5
	i	2.77	0.11	0.064	2.72	7
AB-3	a	1.99	0.17	0.28	29.68	7
	b	2.15	0.17	0.173	22.14	6
	c	2.31	0.18	0.102	15.74	5.5
	d	2.46	0.18	0.064	10.53	4
	e	2.59	0.19	0.061	9.12	6
	f	2.68	0.19	0.071	9.46	8

- denotes data not available

Table 2 - Transfer Function Zeros and Subjective Ratings for Group AB.
(Data obtained and derived from Chalk 1969 and Chalk 1973)

	BB-1	BB-2	BB-3
ω_{dr} (rad/sec)	2.49	2.49	2.60
ζ_{dr}	0.1	0.1	0.1
$ \phi/\beta _{dr}$	5	5	5
$\angle(\phi/\beta)_{dr}$ (degrees)	42	45	46
τ_{roll} (seconds)	0.4	0.4	0.4
τ_{spiral} (seconds)	1000	1000	1000

Table 3 - Modal Data for Group BB.
(Data obtained and derived from Chalk 1969 and Chalk 1973)

Sub-Group	Config.	ω_ϕ (rad/sec)	ζ_ϕ	(p_{osc}/p_{avg})	$(\Delta\beta_{max}/k_\beta)$ (degrees)	Cooper Rating
BB-1	a	1.20	0.08	2.01	18.88	7
	b	1.81	0.07	0.412	9.03	4
	c	1.81	0.07	0.412	9.03	6.5
	d	1.89	0.07	0.321	7.76	7
	e	2.12	0.07	0.144	4.58	4
	f	2.37	0.06	0.033	1.17	5
	g	2.37	0.06	0.033	1.17	6
	h	2.83	0.07	0.082	3.85	8
BB-2	a	1.64	0.08	0.649	12.67	6.5
	b	2.00	0.08	0.224	6.77	3
	c	2.34	0.09	0.039	2.18	2
	d	2.67	0.10	0.044	1.58	4
	e	2.95	0.11	0.107	4.53	7
BB-3	a	0.61	0.13	226.5	30.69	7.5
	b	1.69	0.17	0.660	12.78	4
	c	2.18	0.18	0.184	6.44	3
	d	2.51	0.20	0.079	3.75	2
	e	-	-	0.097	3.74	4
	f	3.15	0.23	0.145	4.70	3.5
	g	3.64	0.26	0.202	6.13	5.5

- denotes data not available

Table 4 - Transfer Function Zeros and Subjective Ratings for Group BB.
(Data obtained and derived from Chalk 1969 and Chalk 1973)

	CB-1	CB-2	CB-3
ω_{dr} (rad/sec)	2.47	2.48	2.44
ζ_{dr}	0.1	0.1	0.1
$ \phi/\beta _{dr}$	high	high	high
$\angle(\phi/\beta)_{dr}$ (degrees)	41	39	46
τ_{roll} (seconds)	0.4	0.4	0.4
τ_{spiral} (seconds)	1000	1000	1000

Table 5 - Modal Data for Group CB.
(Data obtained and derived from Chalk 1969 and Chalk 1973)

Sub-Group	Config.	ω_ϕ (rad/sec)	ζ_ϕ	(p_{osc}/p_{avg})	$(\Delta\beta_{max}/k\beta)$ (degrees)	Cooper Rating
CB-1	a	-	-	-1.697	32.45	9
	b	1.03	0.06	3.376	9.39	7
	c	2.14	0.03	0.127	1.53	6.5
	d	2.49	0.003	0.048	1.06	7
	e	-	-	0.110	2.80	9
CB-2	a	-	-	-1.366	43.92	10
	b	0.69	0.26	13.58	9.70	5.5
	c	2.09	0.12	0.158	2.45	5.5
	d	2.84	0.11	0.086	1.36	8
CB-3	a	-	-	-3.014	16.59	7.5
	b	1.72	0.16	0.482	4.13	6
	c	2.74	0.21	0.113	1.98	5.5
	d	3.26	0.25	0.192	2.45	8
	e	3.88	0.29	0.251	2.96	8

- denotes data not available

Table 6 - Transfer Function Zeros and Subjective Ratings for Group CB.
(Data obtained and derived from Chalk 1969 and Chalk 1973)

$ \phi/\beta _{dr}$	$\angle(\phi/\beta)_{dr}$ (degrees)		
	0	60	120
0	✓	✗	✗
1.5	✓	✓	✓
3.0	✓	✓	✓

✓ denotes tested configuration
✗ denotes not tested

Table 7 - Costigan-Calico Test Matrix.
(Data from Costigan and Calico 1989)

	State Vector	Roll Mode	Spiral Mode	Dutch Roll Mode
Eigenvalue		-4.0	-0.025	(3.0 , 0.4)*
Eigenvector	β	0	0	β_{dr}
	p	1	x	x
	r	0	x	x
	ϕ	x	1	ϕ_{dr}

* (•,•) denotes (frequency (rad/sec),damping) for eigenvalues

x denotes elements not weighted in the cost function

Table 8 - Costigan-Calico Design Parameters.
(Data from Costigan and Calico 1989)

$ \phi/\beta _{dr}$	Yaw Pointing Task		Bank Angle Tracking Task	
	$\angle(\phi/\beta)_{dr}$ (degrees)		$\angle(\phi/\beta)_{dr}$ (degrees)	
	0	120	0	120
0	6.2	-	4.2	-
1.5	5.2	5.7	4.0	4.5
3.0	4.2	6.0	3.8	4.7

Table 9 - Costigan-Calico Tracking Task Average Cooper-Harper Ratings.
(Data from Costigan and Calico 1989)